

MATHEMATICS



For Preparatory Year Two
First Term
Student's Book



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بسم الله الرحمن الرحيم

Dear students:

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It is extremely great pleasure to introduce the mathematics book for second preparatory. We have been specially cautious to make your learning to the mathematics enjoyable and useful since it has many practical applications in real life as well as in the other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate the mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining the patterns of positive thinking which pave your way to creativity.

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration.

Our great interest here is to help you get the information by your self in order to develop your self-study skills.

Calculators and computer sets are used when there's a need for. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

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The used Mathematical Symbols

N	The set of natural numbers	Т	perpendicular to			
Z	The set of integer numbers	//	parallel to			
Q	The set of rational numbers	AB	Line segment AB			
Q,	The set of irrational numbers	AB	Ray AB			
R	The set of real numbers	ĀB	straight line AB			
\sqrt{a}	Square root of number a	m (∠ L)	measure of angle L			
∛a	Cube root of number a	~	Similarity			
[a , b]	Closed interval	<	less than			
]a,b[Open interval	S	less than or equal to			
[a , b[Half-open (closed) interval	>	greater than			
]a , b]	Half-open (closed) interval	2	greater than or equal to			
]-∞,a] [a,∞[Infinite interval	P(E)	probability of occurring event (E)			
Ξ	is congruent to					



Real Numbers



Revision

Think and Discuss

The sets of numbers

The set of Counting numbers = {1, 2, 3, ...}

The set of Natural numbers: N = {0, 1, 2, 3, ...} = counting numbers u {0}

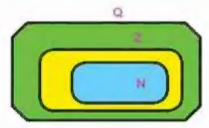
The set of integers : $Z = \{..., -3, -2, -1, 0, 1, 2, 3,...\}$

The set of Positive integers Z* = { 1, 2, 3, ...} = Counting numbers

The set of Negative integers Z = {-1, -2, -3, ...}

Z = Z* U { 0 } U Z-

The set of Rational numbers $Q = \{ -\frac{b}{b} : a, b \in \mathbb{Z}, b \neq 0 \}$



NCZCQ

The absolute value of a rational number:

$$|-7|=7$$
, $|3|=3$, $|0|=0$, $|-\frac{6}{3}|=\frac{6}{3}$

If |a| = 5 then $a = \pm 5$

The Standard form of a rational number is:

$$a \times 10^n$$
 where $n \in z$, $1 \le |a| < 10$

For example:- The standard form of the number
$$25.32 \times 10^4$$

= 2.532×10^5

- The standard form of the number $0.00053 = 5.3 \times 10^{-4}$

The perfect square rational number:

It is that positive number which can be written in the form of a square rational number i.e (rational number)²

Example 1, 4, 25,
$$\frac{9}{16}$$
, $2\frac{1}{4}$, ...

The perfect cube of rational number:

It is that rational number which can be written in the form of a cube rational number. I.e (rational number)³

Example 1, 8, -27, -216,
$$\frac{8}{125}$$
, ...

The square root of a perfect square rational number

- O The square root of the positive rational number a is that number whose square is equal to a.
- (√zero = zero) the square root of zero is zero.
- O Every perfect square rational number a has two square roots each one of them is an additive inverse to the other i.e. \sqrt{a} , $-\sqrt{a}$

Example
$$\frac{16}{25}$$
 has two square roots: $\frac{4}{5}$, $-\frac{4}{5}$

 \bigcirc $\sqrt{9}$ means the positive square root of 9 which is equal to 3

$$\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right| \text{ i.e}$$

$$\sqrt{\left(-7\right)^2} = \left|-7\right| = 7$$



Complete the following table

Number	Natural Number	Integer	Rational Number
3	1	1	1
-3			
3 5			
9			
√ 16			
5 - 7			

Revision exercises

Complete Write the following numbers in the form a where a and b are two integers and there are n't common factors between them, b ≠ zero.

A
$$0.2 = \dots$$
 B $0.3 = \dots$ **C** $25\% = \dots$ **D** $[-0.75] = \dots$ **E** $-6 = \dots$ **E** $1\frac{1}{4} = \dots$

- 2 Choose the correct answer:
 - A The solution set of the equation x + 5 = |-5| in N is ((0) or (10) or (-10), \emptyset)

 - C The product of the rational number a by its additive inverse is

$$(zero, -\frac{a}{b} \text{ or } \frac{a^2}{b^2} \text{ or } \frac{-a^2}{b^2})$$

$$(zero, |-12|, -12|, 6)$$

$$\sqrt{a^2} = \dots$$
 (a or -a or |a| or ±a)

3 Find the value of x which satisfies each of the following equations. Determine whether this value is a natural, real or rational number.

$$\triangle$$
 5X + 3 = 20

B
$$7X + 11 = 12$$

G
$$3X + 5 = 1$$
 D $X + 3 = 7$

$$D X + 3 = 7$$

Find in the simplest form:

B The standard form of 0 00015 is

$$D 2^0 + 2^1 + 2^2 + 2^3 = \dots$$

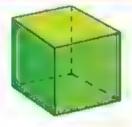
The sum of the two square roots of the number
$$2\frac{1}{4} = \dots$$

The cube root of a rational number



Think and Discuss

you have learned that: The volume of a cube = the length of its side × itself × itself



125

25

5

5

5

5

Complete

The volume of the cube whose side length is equal to 7 cm. = × × = cm³



Let's think

If we have a cube of volume 125 cm³, what is the length of its side?

We search for any three equal numbers of a product equal to 125. Then the number 125 can be factorized into its prime factors

$$125 = 5 \times 5 \times 5$$

... the cube of volume 125cm3 has a side length = 5cm Therefore, 5 is called the cube root of 125 and it is written as $\sqrt[4]{125} = 5$.

The cube root of the rational number a is that number whose cube is equal to a

- The cube root for the rational number a is symbolized by 3/a
- The cube root for a positive rational number is also positive Ex: √125 = 5
- The cube root for a negative rational number is also negative. Ex: ²√-8 = -2 why?
- √zero = zero
- $\sqrt{a^3} = a$

you will learn how

- To find the cube root of a rational number using facorization.
- : To find the cube root of a rational number using the calculator.
- To solve equations that include finding the cube root.
 - To solve applications on the cube root of a rational number.

Key term/

Cube root.



- The number can be factorized into its prime factors...
- A calculator can be used.

The same of the same of

Remark

The perfect cube rational number has one cube root which is also a rational number, why?





Examples

Use factorization to find the value of each $\sqrt[3]{1000}$, $\sqrt[3]{216}$, $\sqrt[3]{3}$; then check your answer using the calculator.

Solution

$$\sqrt[3]{1000} = 5 \times 2 = 10$$

$$\sqrt[3]{1000} = 5 \times 2 = 10$$
 $\sqrt[3]{-216} = -2 \times 3 = -6$

$$3\frac{3}{8} = \frac{27}{8}$$
 3 27 2 8 3 9 2 4 3 3 2 2 1 1

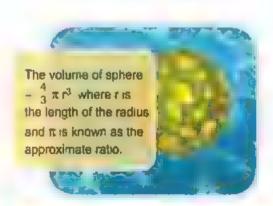
$$\sqrt{3\frac{3}{8}} = \sqrt{\frac{27}{8}} = \frac{3}{2}$$

Use your calculator to check your answer by pressing on

2 Find the length of the radius of a sphere whose volume is equal to 4851cm³ ($\pi = \frac{22}{7}$)

Solution

The volume of the sphere =
$$\frac{4}{3}\pi r^3$$
 3 3087
4851 = $\frac{4}{3} \times \frac{22}{7} r^3$ 3 1029
 $r^3 = \frac{4851 \times 3 \times 7}{4 \times 22} = \frac{9261}{8}$ 7 343
 $\therefore r^3 = \frac{3^3 \times 7^3}{2^3}$ 7 7 7 1



Unit 1: Lesson 1

$$r = \frac{3 \times 7}{2} = \frac{21}{2} = 10.5$$
 cm

we can use the calculator to find $\sqrt[4]{\frac{9261}{\alpha}}$ directly.



Find the diameter of the sphere whose volume is 113.04 cm³ (π = 3.14)



Example

Solve each of the following equations in Q.

A
$$x^3 = 8$$

$$8 x^3 + 9 = 8$$

$$(x-2)^3 = 125$$

Solution

$$\triangle x^1 = 8$$

$$x = \sqrt[3]{8} = 2$$

$$B_{\nu} x^3 + 9 = 8$$

$$x^3 = 8 - 9$$

$$x^3 = -1$$

$$x = \sqrt[3]{-1} = -$$

$$(x-2)^3 = 125$$

$$x - 2 = \sqrt[3]{125}$$

$$x - 2 = 5$$

$$x = 7$$

Solution set
$$= \{7\}$$

$$D_{c} (2x-1)^3 -10 = 54$$

$$(2x-1)^3 = 64$$

$$2x - 1 = \sqrt{64}$$

$$2x - 1 = 4$$

$$2x = 5$$

$$x = \frac{5}{2}$$

Solution set= $\{\frac{5}{2}\}$



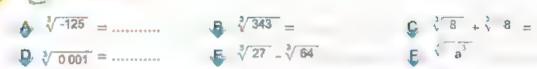
Solve the following equations in Q: $(x + 1)^3 = 27$, $(x + 1)^3 = -27$

Exercises (1-1)

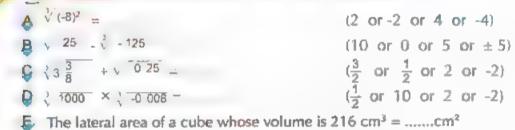
Fill in the following table:

Number a 8 125 -27 3 3 - 8 125 -10 6 -4

Complete



3 Choose the correct answer from the Parentheses:



(36 or 6 or 144 or 216)

$$5\sqrt[3]{x^6} = \sqrt{(x^3 \text{ or } x^2 \text{ or } x \text{ or } x^4)}$$

4 Find the value of x in each of following cases:

$$\triangle \sqrt[3]{x} = 5$$
 $\triangle \sqrt[3]{x} = -\frac{1}{2}$
 $\triangle \sqrt[3]{x} = -\sqrt{4}$
 $\triangle x^3 = -8$
 $\triangle x^3 - 125 = 0$
 $\triangle x^2 = 64$

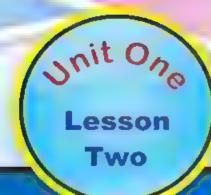
Find the solution set for each of the following equations in Q:

$$x^3 + 27 = 0$$
 B $8x^3 + 7 = 8$ C $(x + 3)^3 = 343$ D $(5x - 2)^3 + 10 = 18$

6 Application problems

- Find the side length of a cube vessel with capacity of one liter.
- Find the diameter length of a sphere whose volume = $\frac{1372}{81}$. π cubic unit (Use $\pi = \frac{22}{7}$) (volume of a sphere = $\frac{4}{3}$. π r^3)

The set of Irrational numbers Q'



Drink and Discuss

you have learned that: A rational number is that number which can be put in the form:

$$\frac{a}{b}$$
: where a , $b \in z$, $b \neq 0$

for example: when solving the equation $4x^2 = 25$

then
$$x^2 - \frac{25}{4}$$
 $\therefore x = \pm \frac{5}{2}$

$$\therefore x = \pm \frac{5}{2}$$

Remark Each of $\frac{5}{2}$, $-\frac{5}{2}$ is a rational number.

However, there are many numbers which can not be put in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$

for example: when solving the equation $X^2 = 2$, we can not find any rational number whose square is egual to 2



irrational numbers

key termi

Simational number



The irrational number

It is that number which can not be put in the form $\frac{a}{b}$ where $a, b \in Z, b \neq 0$



the following are examples to irrational numbers.

First: the square roots of the positive numbers which are not perfect squares

Ex:
$$\sqrt{2}$$
, $\sqrt{5}$, $-\sqrt{6}$, $\sqrt{7}$

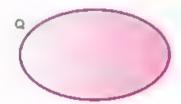
Second: the cube roots of those numbers that are not perfect cubes

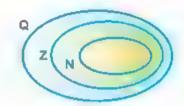
Ex:
$$\sqrt{4}$$
, $\sqrt{-2}$, $\sqrt{11}$, ...

Third: the pi π (the approximation ratio)

Where it is impossible to find any exact value for any of the previous number, why?

Those numbers and others form a set which is called the set of irrational numbers which is denoted by the symbol Q'.





$$Q \cap Q' = \emptyset$$



Think: is √-1 an irrational number? why?

Exercises (1-2)

Complete :using the symbol Q or Q`.

Put (✔) on the true sentence and (X) on the false sentence

$$\triangle$$
 2.3 × 10⁵ ∈ Q (

$$\triangle$$
 2.3 × 10⁵ \in Q () \triangle | -5 | \in Q ()

$$Q \sqrt[3]{-4} \in Q^*$$
 () $E \sqrt{1000} \in Q$ () $E \sqrt{7} > 3$ ()

G
$$\sqrt[4]{10} > 2$$
 () **H** $\sqrt[4]{20} > \sqrt{9}$ ()

- The side length of a square whose surface area = 6cm² is a rational number ()
- Choose the correct answer:
 - A. The surface area of a square whose side length is $\sqrt{3} = ...$

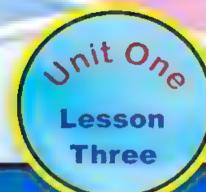
B The irrational number lies between 3 and 4 is

$$(3.5 \text{ or } \frac{1}{8} \text{ or } \sqrt{13} \text{ or } \sqrt{20})$$

The irrational number lies between -2 and -1 is

$$(-3 \text{ or } -1 \frac{1}{2} \text{ or } -\sqrt{3} \text{ or } \sqrt{2})$$

Finding the approximate value of an Irrational number



Think and discuss

Can you find the two rational numbers which the irrational number \(\sqrt{2} \) is located between them.

Romark

$$\sqrt{2}$$
 is between $\sqrt{1}$, $\sqrt{4}$ i.e $1 < \sqrt{2} < 2$
i.e. $\sqrt{2} = 1 + a$ decimal fraction

To find the approximate value of \$\sqrt{2}\$ We check the values of the following numbers:

$$(1.1)^2 = 1.21$$
, $(1.2)^2 = 1.44$, $(1.3)^2 = 1.69$,

$$(1.4)^2 = 1.96$$
, $(1.5)^2 = 2.25$

$$1.4 < \sqrt{2} < 1.5$$

i.e.
$$\sqrt{2} = 1.4 + a$$
 decimal fraction

i.e.
$$1.41 < \sqrt{2} < 1.42$$

Use the calculator to check you answer.



You will learn how

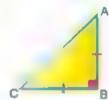
- To find the approximate value for an irrational number
- To represent an irrational number on the number line.
- & To solve equations in O

Representing the irrational number on the number line.

How can the point represents \ 2 be located on the number line?

If we draw the right triangle ABC at B which is an isosceles triangle also. where AB = BC = one unit of length Then $(Ac)^2 = (AB)^2 + (BC)^2 = 1^2 + 1^2 - 2$

 $AC = \sqrt{2}$ unit of length.



-) draw the number line and place the sharp point of the compasses at point O, then adjust the compasses to a length that is equal to \overline{AC} and draw an arc that intersects the number line on the right of \overline{O} and at the point X, where that point represents $\sqrt{2}$
-) Using the same length, we can label the point X' which represent $-\sqrt{2}$ where X' is on the left of the point ϕ .

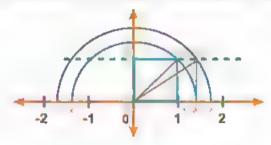


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Think : Label the point which represents $3 + \sqrt{2}$ on the number line.



Activity · Draw the square OABC whose side length is equal to one unit of length.



The length of its diagonal = $\sqrt{1+1} = \sqrt{2}$ unit of length

$$\therefore OB = \sqrt{2}$$

-) Place the sharp point of the compasses at point O and draw a semi-circle whose diameter = the length of $\overline{OB} = \sqrt{2}$.
-) OA \cap the semi-circle = {X , X`} where X represents the number $\sqrt{2}$, x` represents the number $-\sqrt{2}$.
- Draw XD // AB and intersects CB at D (OD)² = (OX)² + (XD)² = $(\sqrt{2})^2$ + $(1)^2$ = 3 ∴ OD = $\sqrt{3}$
- Place the sharp point of the compasses at point O and adjust it to a length which is equal to the length of OD, then draw semi-circle that intersects with OA at points Y, Y*
- \therefore OY = $\sqrt{3}$ i.e. point Y represents $\sqrt{3}$, while point Y represents $\sqrt{3}$
- O Continue using the same method to represent $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, . .

also
$$-\sqrt{4}$$
, $-\sqrt{5}$, $-\sqrt{6}$,...







- Two consecutive integers that $\sqrt{5}$ lies between them.
- Two consecutive integers that $\sqrt{12}$ lies between them.
- Two consecutive integers that √10 lies between them.
- Two consecutive integers that ₹ -20 lies between them.



2 Prove that:

- \triangle $\sqrt{3}$ lies between 1.7.1.8.
- B √15 lies between 2.4, 2.5.
- 3 Find the value of $\sqrt{11}$ to the nearest hundredth.
- Find the value of ³√2 to the nearest tenth.
- Draw the number line and label the point which represents the irrational number $\sqrt{3}$.
- 6 Draw the number line and label the point which represents the irrational number $1 + \sqrt{2}$



Example (1)



Find the solutions set for each of the following equations in Q:

$$A x^2 = 2$$

$$x^3 = 5$$

$$\frac{4}{3} x^2 = 1$$

Solution

$$A x^2 = 2$$

$$\therefore x = \pm \sqrt{2}$$
 Solution set = $\{-\sqrt{2}, \sqrt{2}\}$

$$B - x^3 = 5$$

$$\therefore x = \sqrt[4]{5}$$
 Solution set = $\{\sqrt[4]{5}\}$

$$C = \frac{4}{3} x^2 = 1$$

$$\therefore \frac{3}{4} \times \frac{4}{3} \times 2 = \frac{3}{4} \times 1$$

$$x^2 = \frac{3}{4}$$

$$\therefore x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$

$$\therefore x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$
 Solution set $= \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$

$$0.001 x^3 = -8$$

$$x^3 = -\frac{8}{0.001} = -8000$$

The solution set in Q'= Ø



Example (2)



Find the length of each of the side and the diagonal of a square whose area is 7cm2.

Solution

Let the length of the side be x cm.

then the area = $x \times x = x^2$

Where L is the square diagonal length

$$\chi^2 = 7$$

$$\therefore x = \pm \sqrt{7} \text{ cm}$$

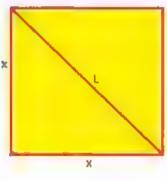
$$\therefore x = \sqrt{7}$$
 cm why?

To find the diagonal of the square: use pythagorean theorem

$$L^2 = x^2 + x^2$$
 Where L is the square diagonal length.

$$\therefore L^2 = 14$$

$$\therefore L = \pm \sqrt{14}$$
 cm $\therefore L = \sqrt{14}$ cm why?



Example (3)



Find: the circumference of a circle whose area is 3 7 cm²

Solution

The area of the circle = πr^2

$$3 \mathcal{R} = \mathcal{R} r^2$$

$$r^2 = 3$$

$$r = \sqrt{3}$$
 cm

$$r = \sqrt{3}$$
 cm or $r = -\sqrt{3}$ cm (refused)

the circumference =
$$2 \pi r = 2 \pi \times \sqrt{3} = 2 \sqrt{3} \pi$$
 cm.

Exercises (1-3)

Circle the irrational number in each of the following:

 $\sqrt{3}$, -0,2, $\sqrt{-1}$, 0, $\sqrt{9}$, - $\sqrt{\frac{4}{26}}$

Find the value of X in each of the following cases and determine whether $x \in Q \text{ or } x \in Q$

 $4x^2 = 9$

B $2x^2 = 6$

 $C_{x^3} = 125$

 $p_{x^3} = 10$

 $E(x-1)^2 = 4$

 $E(x-2)^3 = 1$

- Find the approximate value for \$10, and check your answer using the calculator.
- Think If x is an integer, find the value of X in each the following cases

 \triangle x < $\sqrt{7}$ < x + 1 \bigcirc x < $\sqrt{80}$ < x + 1 \bigcirc x < $\sqrt{125}$ < x + 1 \bigcirc x < $\sqrt{5}$ < x + 1 \bigcirc x < $\sqrt{30}$ < x + 1 \bigcirc x < $\sqrt{100}$ < x + 1

Choose the correct answer:

A the irrational number lies between 2 and 3 is. (v 10 or v 7 or 25 or v 3.)

a √10 ~

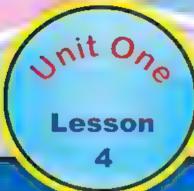
(2.99 or 3.71 or 3 or - 3.2).

The nearest integer to \$25 is . . (5 or 3 or 2 or 12.5)

The square whose area is 10 cm², its side length is cm

(5 or -5 or √10 or - √10).

- The cube whose volume is 64 cm³ its side length is . . . cm (8 or 4 or 16 or 64).
- Draw the number line and label point A which represents > 2 and Label point B which represents $1 + \sqrt{2}$, Label point C which represents 1 - √2
- Draw the right triangle ABC at B where AB = 2cm, BC = 3cm, then use the figure number line.



The set of the Real numbers R

Think and Discuss

You will learn how

- To define the set of real numbers (R)
- % To define the realtion among sets of N, Z, Q. Q', R

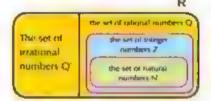
Key terms

A real number.

You have learned the set of rational numbers (Q), you have also found that there are other numbers that form the set of irrational number Q\ such as $\sqrt{2}$, $\sqrt[3]{2}$, π ,... However, the union of these two sets forms a new set called the set of the real numbers, and it is denoted by the symbol R

Look at the opposite Venn diagram, you find that:

- R ≠ Q ∪ Q;
- Any natural, integer, rational or irrational number is a real number



NeZeQeR and so is Q' c R

- Think Give examples from your own to some real numbers which are rational or irrational numbers.
- Every real number is represented by one point on the number line.

Negative real numbers O Positive real numbers

First: zero is represented by the origin O.

Second: the positive real numbers are represented by all the points On the number line that are located on the right side of O

Third: the negative real numbers are represented by all the points on the number line that are located on the left side of O



Put each of the following numbers in its suitable place on the opposite venn diagram.

	41	(†) i	91	R	μι	40	-	OF	I U	116	ob	pos	2116	¥	er	ИП	• I	agi	Ç.	П
1 2	,	-4	J	9	r	V	5	,	0,0	6,	7 9	, ∛	-2	p	V	16	,	0	, .	5

Label point A on the number line which represents $\sqrt[3]{-8}$, and point B which represents $\sqrt{9}$, then find the length of $\overline{A8}$.

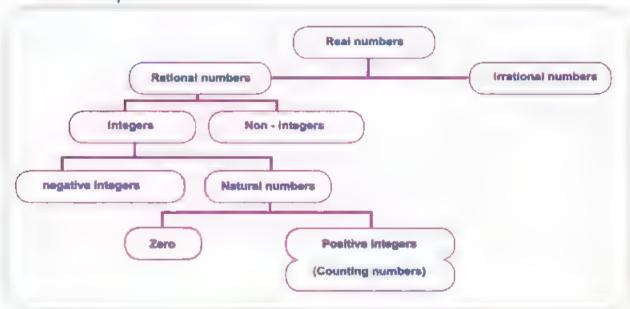


- State if each sentence is true or false:
 - Every natural number is a positive real number.
 - B Every integer is a real number.

$$\sqrt[4]{-1} = -1$$
 because -1 x -1 x -1 = -1

While $\sqrt{-1} \notin \mathbb{R}$ because there is no real number if multiplyed by it self, the product is -1.

Q'



Discuss with your teacher and classmates: Are there any non-Real number?

Exercises (1 - 4)

Study the previous chart and answer the following by placing (V) on the true sentence and (X) on the false sentence:

4	Every natural number is an integer.)
8	Zero ∈ The set of rational numbers	-()
Ç	Z = Z* u Z*	- (- }
D.	Any non-integer number is a rational number	4	- 1

2 Complete the following table by placing (✔) in the suitable place as shown in the first case:

Number	natural number	integer number	rational number	irrational number	real
-5	х	1	/	×	1
√2					
1 1 2					
₹6					
[-2]					
~ v 4					
5 2					
0,3					
v 1					

Ordering numbers at R



Think and Discuss

If A, B are two points that belong to the straight line L, and we determined a certain direction as shown by the arrow; then we can say that:



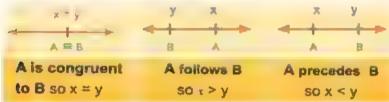
- The point 8 follows the point A. Le on its right hand side.
- O The point A precedes the point B. i.e on its left hand side.

The same applies for all the points on the straight line. However, if we know that every point on the straight line represent a real number. We can say that:

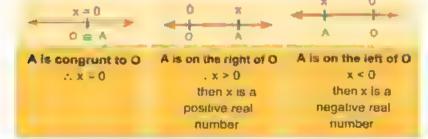
the set of real number is an ordered set.:

The properties of order:

If c, y are two real numbers represented on the number line by the two points A, B respectively, the ordering relation can be one of the following three cases:



If x is a real number represented by the point A on the number line while O is the origin point which represents the zero, then the ordering relation can be one of the following three cases.



You will learn how

To define the ordering relation in R

Key term/

- ordering relation
- 5 more than
- Less than
- Sequal to
- Ascending order
- Descending order.

The set of the positive real numbers: $R^* = \{x : x \in \mathbb{R}, x > 0\}$ The set of the nagative real numbers: $R' = \{x : x \in \mathbb{R}, x < 0\}$

The set of non-negative real numbers = $R^* \cup \{0\} = \{x : x \ge 0, x \in R\}$ The set of the non - positive real numbers = $R \cup \{0\} = \{x : x \le 0, x \in R\}$



Example:

Arrange the following numbers ascendingly $\sqrt{27}$, - $\sqrt{45}$, $\sqrt{20}$, 6, 0, $\sqrt[3]{-1}$

Solution

$$6 = \sqrt{36}$$
, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

The ascending order is from the smallest to the greatest.

$$-\sqrt{45}$$
, $-\sqrt{1}$, 0, $\sqrt{20}$, $\sqrt{27}$, $\sqrt{36}$
i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0, $\sqrt{20}$, $\sqrt{27}$, 6.

Exercises (1-5)

- 1. Arrange the following number descendingly: $\sqrt{62}$, 8, - $\sqrt{50}$, $\sqrt{70}$
- If $x \in R$, state whether x is a positive or negative or anything else in each of the following.

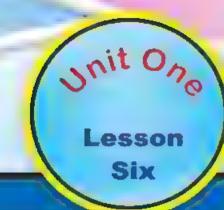
$$A \times > 0$$

$$B_{\nu} x < 0$$

- Prove that $\sqrt{3}$ lies between 1.7, 1.8, then represent $\sqrt{3}$, 1.7 and 1.8 on the number line.
- Find the side length in a square whose area is 5 cm², is the side length a rational number?
- Find the side length in a cube whose volume is 1.728 cm3, is the side length a rational number?
- Put the suitable notation(> or < or =)

$$D = 1 + \sqrt{2}$$





Think and Discuss

Interval is a subset of the set of real numbers

first: the limited intervals

If a, b ∈ R, a < b, then we can define each of:

The closed inteval

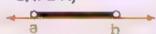
$$[a, b] = \{x : a \le x \le b, x \in R\}$$

[a, b] c R in which the elements are a, b and all the real numbers between them.

When we draw that interval, we put a shaded circle at each of the two points a and b then, we shade that area between them on the number line.

The open interval

]a, b [=
$$\{x: a < x < b, x \in R\}$$



la, bl c R in which the elements are all the real numbers between the two numbers a, b

When we draw that interval, we put an unshaded circle at each of the two points which represent the two numbers a and b then, we shade that area between them on the number line.

Practice

Write down each of [3,5], [3,5] using the description method then represent them on the number line.

You will learn how

- To define limited
- To define unlimited intervals.
- To recognize the operations on intervals.

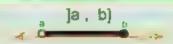
key term/

- Limited interval
- dosed interval
- open interval
- half- open interval
- unlimited interval
- noinu 🤣
- Intersection
- difference
- 5 complement

Half openor (half closed) intervals

 $[a,b] = \{x: a \le x < b, x \in R\}$

(a, b) c R where its elements are the number a and all the numbers between a and b.



 $[a,b] = \{x: a \le x \le b, x \in R\}$

[a , b] c R where its elements are the number b and all the number between a and b.



the way

Write down each of the two intervals: [3 , 5[,]3, 5] using the description method, then represent them on the number line.



Examples:

Represent each of the following intervals on the number line: [-1, 4] ,]-1, 4[,]-1, 4], {-1, 4}

Solution



Discuss with your teacher and your classmates whether the interval is a finite or an infinite set.



Write down the following sets in the form of intervals, then represent them on the number line:

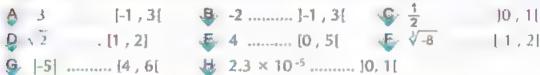
$$\mathcal{G}$$
 $X = \{x : 0 \le x \le 4, x \in \mathbb{R}\}$

$$B_{y} X = \{x: -2 \le x < 3, x \in R\}$$

$$D_{x} X = \{x: -3 < x \le -1, x \in R\}$$











Second: The unlimited intervals

You know that: If the number line of real numbers is expanded on its two direction, we get more positive real numbers at the right direction and more negative real number at the left direction such all those numbers are located on that line.

- The symbol (∞) is read (infinity) and it is more than any imagined real number, ∞ ∈ R
-) The symbol (-∞) is read (negative infinity) and it is less then any imagined real number, -∞ ∉ R
- The two symbols ∞, ∞ can not be represented by any points on the number line and they are expansions to the number line at its two directions.



If a is a real number, then we can define the following unlimited intervals:

 $[a, \infty[= \{ x : x \ge a, x \in R \}]$

a a

The interval [a , ∞[

That interval represents the number a and all the real numbers which are more than a

The interval [-∞ , a]

$$[-\infty, a] = \{x : x \le a, x \in R\}$$



That interval represents the number a and all the real number which are less than a.



Write down each of the following intervals $[3, \infty]$, $]-\infty$, $[3, \infty]$ using the description method, then represent them on the number line.

the interval la. ...[

$$a,\infty[=\{x:x>a,x\in R\}$$

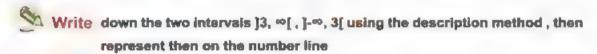
That interval represents all the real number which are more them a

the interval]-. a[

$$[-\infty, a] = \{x : x < a, x \in R\}$$



that interval represents all the real numbers which are less than a





Write down the following sets in the form of intervals, then represent them on the number line.

$$A = \{x : x \ge 2, x \in R\}$$

$$C X = \{x: x \ge -7, x \in R\}$$

$$\sum_{\mathbf{D}} \mathbf{X} = \{\mathbf{x} : \mathbf{x} \leqslant \sqrt[4]{-8}, \mathbf{x} \in \mathbf{R}\}$$

Put the suitable symbol ∈ or ∉ or ⊂ or ⊄ To make each statement true:

Operations on intervals

Since all the intervals are subsets of the set of the real number R, The operations of union, intersection, difference and complement can be applied on the intervals. The graphical representation to the intervals on the number line contributes to determine and verify the result of any operation. This can be clarified from the following examples.



Examples

1 If X = [-2, 3], Y = [1, 5], find the following using the number line:

Solution

B
$$\times$$
 u Y = [-2, 3] \cup [1, 5] = [-2, 5]



A M -J

- **B** MnJ
- C MuJ

- D J ∪ {2, 3}
- & M
- ₽ J'

Solution

$$A M - J = [2, \infty] - [-2, 3] = [3, \infty]$$

$$C = M \cup J = [2, \infty] \cup J - 2, 3 [=] - 2, \infty[$$

D
$$\{0, \{2, 3\} = [-2, 3] \cup \{2, 3\} = [-2, 3]$$

$$E = [-\infty, 2]$$

$$[\mathbf{b}] =]-\infty, -2] \cup [3, \infty[$$



Practice

Put (/) on the true sentence and (x) on the false sentence:

$$\triangle$$
 [-2, 5] - (2, 5) = [-2, 5]

$$\mathbb{P}$$
 [-1, 3] n [1, 4] = [1, 3]

B.
$$[-1, 3] \cup (-1, 0) = [-1, 0]$$

$$C$$
 [2, 5] - {5} = [2, 5]

Exercises (1-6)

Complete the following table as shown in the first example:

interval	Representation by using the descri-tion method	Graphical representation on the number line
[-1 , 2]	$\{x -1 \leqslant x \leqslant 2, x \in R\}$	-1 0 1 2
[1,3[
]-∞ , 2]		
	$\{x: 0 \le x \le 3, x \in \mathbb{R}\}$	
	$\{x : x \ge -1, x \in R\}$	
		-2 -1 0 1 2 3
		- 00
]1, 5[
	$\{x : x \ge 0, x \in R\}$	

Complete using ∈ or ∉ A 3 [2, 3] D (1)

Ly 1[E [-2] [2, y

the same of the sa

Choose the correct answer:

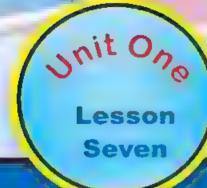
$$A[2,7]-\{2,7\}=...$$

$$[0,5] \cup [3,8[=(]3,5] \text{ or } [3,5] \text{ or } [0,8] \text{ or } [0,8[)$$

$$D = [-1, 2[-[1, 4] = (]-1, 1[or {-1, 1} or]-1, 1] or [-1, 1])$$

If X = [-1, 4], $Y = [3, \infty[, Z = \{3, 4\}]$, find each of the following using the number line:

Operations on the real numbers

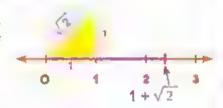


Hink and Discuss

First: The properties of adding the real numbers :

You have determined the location of the point X which represents the number $1 + \sqrt{2}$ on the number line. Since it represents the sum of the two real numbers 1 and v 2 then the sum of every two real numbers is a real number.

i.e, the set of the real numbers R is closed under the operation of addition.



the closure property

Ifa ← R, b ∈ R then (a + b) ∈ R

for example : each of 2 + 3, 1 + $\sqrt{2}$, -2 + $\sqrt{5}$ and 2 + $\sqrt[3]{3}$ are real numbers.

The commutative If $a \in R$, $b \in R$ then a + b = b + a

for example : $2 + \sqrt{3} = \sqrt{3} + 2$, $3 - \sqrt{5} = -\sqrt{5} + 3$

property

The associative If a ∈ R, b ∈ R, c ∈ R,

then (a+b)+c=a+(b+c)=a+b+c

for example: $(3 + \sqrt{2}) + 5 = 3 + (\sqrt{2} + 5)$ associative property = $3 + (5 + \sqrt{2})$ commutative property $= 3 + 5 + \sqrt{2}$ associative property $= B + \sqrt{2}$

You will lears how

- To solve operations on the real numbers.
- To define the properties of operations on the real numbers .

key terms

- Closure property.
- Commutative property.
- & associative property.
- Additive neutral.
- Additive Inverse.
- multiplicative neutral.
- multiplicative inverse.
- b notibution of multiplication on addition or aubtraction.

Zero is the additive neutral element $f(a) \cdot R$ then a + 0 = 0 + a = a

for example:
$$\sqrt{5} + 0 = 0 + \sqrt{5} = \sqrt{5}$$
, $-\sqrt[3]{4} + 0 = 0 + (-\sqrt[3]{4}) = -\sqrt[3]{4}$

Each real number has an additive inverse

For a number a ∈ R there is ⊕a∈ R where a + (-a) = (-a) + a = zero

for example $\sqrt{3} \in \mathbb{R}$, has additive inverse $(-\sqrt{3}) \in \mathbb{R}$ where $\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} = zero.$



Complete the following to have a true sentence:

$$\triangle \sqrt{2} + 5 = 5 +$$

$$\sqrt{11} + (-\sqrt{11}) = .$$

$$\bigcirc$$
 7 + $\sqrt{3}$ = 5 + (.....+.....+.....)

$$\blacksquare$$
 the additive inverse for $(1 - \sqrt{2})$ is

$$\sqrt{5} \sqrt{3} + (-\sqrt{3}) = ...$$

G
$$7 + \sqrt{5} - 3 =$$

$$\mathbf{H} = (4 + \sqrt{7}) + (3 - \sqrt{7}) =$$

If
$$a \in R$$
, $b \in R$, then $a \cdot b$ means the sum of the number a and _____ of the number b .

J If a ∈ N, b ∈ Q, c ∈ R, then
$$(a + b + c)$$
 ∈

Discuss the following with your teacher and classmates, then give examples:

- A Is subtraction a commutative operation in R?
- Is subtraction an associative operation in R?

Second: The properties of multiplying the real numbers

The closure property

If a & R, b & R then a × b & R

the set of real number is closed under the operation of multiplication. i.e the product of multiplying every two real number is a real number

for example:
$$5 \times \sqrt{2} = 5\sqrt{2} \in \mathbb{R}$$
, $\sqrt{3} \times \sqrt{3} = 3 \in \mathbb{R}$
 $-2 \times \sqrt[3]{5} = -2\sqrt[3]{5} \in \mathbb{R}$, $\frac{2}{3} \times \pi = \frac{2}{3}\pi \in \mathbb{R}$
 $2\sqrt{3} \times \sqrt{3} = 6 \in \mathbb{R}$, $2\sqrt{3} \times 5 = 10\sqrt{3} \in \mathbb{R}$

Commutative property \(\) If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then a, b = b, a

for example :
$$\sqrt{2} \times 3 = 3 \times \sqrt{2} = 3\sqrt{2}$$

$$(a.b).c = a.(b.c) = a.b.c$$

for example:
$$\sqrt{2} \times (5 \times \sqrt{2}) = (\sqrt{2} \times 5) \times \sqrt{2} = (5 \times \sqrt{2}) \times \sqrt{2}$$

= $5 \times \sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

One is the multiplicative neutral

for example :
$$2\sqrt{5} \times 1 = 1 \times 2\sqrt{5} = 2\sqrt{5}$$

Every real number > 0 bits a multiplicative inverse.

It exist an real number - such that

a 1 a 1 (1 is the neutral element of multiplication)

for example: the multiplicative inverse for $\frac{\sqrt{3}}{2}$ is $\frac{2}{\sqrt{2}}$

where
$$\frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

Remarks:
$$\frac{a}{b} = a \times \frac{1}{b}$$
, $b \neq 0$

i.e.
$$\frac{a}{b} = a \times \text{the multiplicative inverse of } b$$
.

Discuss with your teacher: is the division operation commutative in R? is the division operation associative in R?



Examples



Write down each of the following numbers $\frac{6}{\sqrt{2}}$, $-\frac{5}{\sqrt{3}}$, $\frac{15}{2\sqrt{5}}$ where the denominator is an integer.

Solution

Note that the multiplicative neutral is 1 and it can be written in the form $\frac{\sqrt{2}}{\sqrt{2}}$ or $\frac{\sqrt{5}}{\sqrt{6}}$ or

$$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = \frac{3\sqrt{2}}{1} = 3\sqrt{2}$$

$$-\frac{5}{\sqrt{3}} = -\frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{5\sqrt{3}}{3}$$

$$\frac{15}{2\sqrt{5}} = \frac{15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{2\sqrt{5}} = \frac{3\sqrt{5}}{2}$$

Practice

Complete the following to have a true sentence:

$$\mathbf{B} = 3 \times \sqrt{5} = \sqrt{5} \times$$

$$D_{\nu} = 2\sqrt{5} \times 3\sqrt{5} =$$

The multiplicative inverse for
$$\frac{3}{\sqrt{2}}$$
 is

Write each of the following numbers such that the denominator is an integer:

$$\frac{8}{3\sqrt{2}}$$

$$Q = \frac{25}{2\sqrt{10}}$$

Distribution of multiplication on addition

For any three real numbers a, b, c.

$$a \times (b + c) = (a \times b) + (a \times c) = ab + ac$$

$$(a + b) \times c = (a \times c) + (b \times c) = ac + bc$$



Examples

- Simplify the following to the simplest form.
 - A 2√5 (3+√5)

$$(\sqrt{2} + 5)(3 + \sqrt{2})$$

\$ (2-3√5)2

Solution

$$2\sqrt{5} (3 + \sqrt{5}) = 2\sqrt{5} \times 3 + 2\sqrt{5} \times \sqrt{5}$$

$$= 2 \times 3 \times \sqrt{5} + 2 \times 5 = 6\sqrt{5} + 10$$

$$\begin{cases} \sqrt{2} + 5 \end{pmatrix} (3 + \sqrt{2}) = \sqrt{2} (3 + \sqrt{2}) + 5 (3 + \sqrt{2}) \\ = \sqrt{2} \times 3 + \sqrt{2} \times \sqrt{2} + 5 \times 3 + 5 \times \sqrt{2} \\ = 3\sqrt{2} + 2 + 15 + 5\sqrt{2} \\ = 3\sqrt{2} + 17 + 5\sqrt{2} = 8\sqrt{2} + 17$$

Give an estimation to the result of $(3 + \sqrt{5}) \times (1 + \sqrt{8})$, then check your answer using the calculator.

Solution

First: The estimate of
$$\sqrt{5}$$
 is 2 \therefore (3 + $\sqrt{5}$) the estimate of 3 + 2 = 5

the estimate of $\sqrt{8}$ is 3 \therefore (1 + $\sqrt{8}$) the estimate of 1 + 3 = 4

$$\therefore (3+\sqrt{5})(1+\sqrt{8})$$
 the estimate of $5 \times 4 = 20$

Second: when we use the calculator to find $(3 + \sqrt{5}) \times (1 + \sqrt{8})$

We find that the result is 20,0459

Therefore, the estimate is reasonable.

Exercises (1-7)

Choose the correct sentence:

$$\triangle 2\sqrt{3} + 3\sqrt{3} = \dots (5\sqrt{6} \text{ or } 5\sqrt{3} \text{ or } 6\sqrt{3} \text{ or } 5\sqrt[3]{3})$$

$$65+7\sqrt{2}-4+\sqrt{2}=......$$
 (15 or 1+7 $\sqrt{2}$ or 1+8 $\sqrt{2}$ or 1+8 $\sqrt{2}$)

$$\bigcirc -2\sqrt{3} \times \sqrt{3} = \dots (-6 \text{ or } -2\sqrt{3} \text{ or } 2\sqrt{3} \text{ or } 8)$$

$$\frac{6}{\sqrt{2}} = (\sqrt{2} \text{ or 2 or 2 } \sqrt{3} \text{ or 6 } \sqrt{3})$$

$$(2\sqrt[3]{5})^3 = \dots (10 \text{ or } 20 \text{ or } 4\sqrt[3]{5} \text{ or } 40)$$

Simplify to the simplest form:

$$\triangle \sqrt{2} (5 + \sqrt{2})$$

$$\sqrt{7} (\sqrt{7} + 2)$$

$$\sqrt{2} (\sqrt{2} + 1)(\sqrt{2} - 1)$$

Write each of the following numbers where the denominator is an integer:

$$\frac{6}{2\sqrt{3}}$$

$$\frac{8}{\sqrt{6}}$$

$$\Phi = \frac{\sqrt{2} + 3}{\sqrt{2}}$$

Simplify to the simplest form:

A
$$2\sqrt{3} + 5 + \sqrt{3} - 6$$

$$(\sqrt{3} + 2)(\sqrt{3} - 1)$$

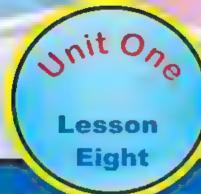
$$\sqrt{5}(3-\sqrt{5})-2(1+\sqrt{5})$$

[5] IF
$$a = \sqrt{3} + 2$$
, $b = \sqrt{3} - 2$ Find the value of each of the following:

(6) If
$$X = \sqrt{15} + 2$$
, $Y = 4 - \sqrt[3]{25}$ estimate the value:

Check the reasonability of each value using your calculator.

Operations on the square roots



Think and Discuss

If a, b are two non-negative real numbers, then

First:
$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

For example:
$$\sqrt{2} \times \sqrt{3} = \sqrt{2} \times 3 = \sqrt{6}$$

$$\sqrt{2} \times \sqrt{10} = \sqrt{2 \times 10} = \sqrt{20}$$

$$\sqrt{15} \times \sqrt{5} = \sqrt{15 \times 5} = \sqrt{73}$$

$$\sqrt{v \cdot a \times b} = \sqrt{a} \times \sqrt{b}$$

For example:
$$\sqrt{20} = 14 \times 5 = 14 \times 15 = 215$$

$$\sqrt{75} = \sqrt{35 \times 3} = \sqrt{25} \times \sqrt{3} = 5 \vee 3$$

Second:

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$
 where $b \neq 0$

For example: $\sqrt{\frac{5}{9}} = \sqrt{\frac{5}{9}} = \frac{1}{3}\sqrt{5}$

$$\sqrt{\frac{16}{3}} = \sqrt{16} = \sqrt{\frac{4}{3}} \times \sqrt{\frac{1}{3}} = \frac{4 \times 3}{3}$$

Third: $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} = 0$

For example :
$$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$\frac{\sqrt{84}}{\sqrt{7}} - \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} - \sqrt{4 \times \sqrt{3}} - 2\sqrt{3}$$

You will learn how

- To conduct operations on the square roots.
- To multiply two conjugates.

key termr

- % Square root.
- Two conjugates numbers



the same of the same of

Examples

Simplify to the simplest form $\sqrt{32} = \sqrt{72} + 6\sqrt{\frac{1}{2}}$

Solution

$$\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}} = \sqrt{16 \times 2} - \sqrt{36 \times 2} + 6 \times \frac{\sqrt{1}}{\sqrt{2}}$$

$$= \sqrt{16} \times \sqrt{2} - \sqrt{36} \times \sqrt{2} + 6 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2} - 6\sqrt{2} + 3\sqrt{2} = \sqrt{2}$$

2 If $x = 2\sqrt{5}$ -1, $Y = 2 + \sqrt{5}$ find the value of $x^2 + y^2$

Solution

$$x^{2} = (2\sqrt{5} - 1)^{2} = (2\sqrt{5})^{2} - 4\sqrt{5} + 1$$

$$= 4 \times 5 - 4\sqrt{5} + 1 = 21 - 4\sqrt{5}$$

$$y^{2} = (2 + \sqrt{5})^{2} = 4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$$

$$x^{2} + y^{2} = 21 - 4\sqrt{5} + 9 + 4\sqrt{5} = 30$$



put each of the following in the form of a vib. where a and biare integers, bis the least possible value:

$$\sqrt{5} \frac{1}{3} \sqrt{162}$$

Simplify to the simplest form:

$$\triangle$$
 $2\sqrt{18}$ \times $3\sqrt{2}$

A
$$2\sqrt{18} \times 3\sqrt{2}$$
 B $\sqrt{5} \times 2\sqrt{10}$ **G** $3\sqrt{7} \times 2\sqrt{28}$

$$1.0 \sqrt{50} + \sqrt{8}$$

$$2 \sqrt{50} + \sqrt{8}$$
 $\sqrt{20} - \sqrt{45}$

$$\sqrt{27} + 5\sqrt{18} - \sqrt{300}$$

Find the value of X + Y, X × Y in each of the following cases:

$$\triangle x = 3 + \sqrt{5}, y = 1 - \sqrt{5}$$

$$\triangle x = 3 + \sqrt{5}$$
, $y = 1 - \sqrt{5}$ $B_1 x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$

$$9 x = 5 - 3\sqrt{2}$$
, $y = 5 - 3\sqrt{2}$

The two conjugate numbers

If a and b are two positive rational numbers.

Then each of the two number [va+vb), va-vb] is a conjugate to the other one.

then, their sum is = $2\sqrt{a}$ twice the first term

and their product is
$$= (\sqrt{a} + \sqrt{b})$$
, $(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 + (\sqrt{b})^2 = (a + b)$

= The square of the first term - The square of the second term

The product of two conjugates is always a rational number

If we have a real number whose denominator is written in the form $(\sqrt{a} \pm \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.





Complete

- $\triangle \sqrt{5} + \sqrt{2}$ their conjugate () and their product is (.)
- By $5 \sqrt{3}$ their conjugate (and their product is (.)
- **G** $2\sqrt{3} + \sqrt{2}$ their conjugate () and their product is (



Examples

Given
$$x = \frac{8}{\sqrt{5} - \sqrt{3}}$$
, $y = \frac{2 - \sqrt{3}}{2 + \sqrt{3}}$



Write both of X and Y where the denominator is a rational number, then find X + Y

Solution

$$x = \frac{8}{\sqrt{5} - \sqrt{3}} = \frac{8}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$$
$$= \frac{8(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^{3} - (\sqrt{3})^{3}} = \frac{8(\sqrt{5} + \sqrt{3})}{5 - 3} = 4(\sqrt{5} + \sqrt{3})$$

$$y = \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$
$$= \frac{(2 - \sqrt{3})^2}{4 + 3} = \frac{4 - 4 + \sqrt{3} + 3}{1} = 7 \cdot 4 + \sqrt{3}$$

$$x + y = 4\sqrt{5} + 4\sqrt{3} + 7 - 4\sqrt{3} = 4\sqrt{5} + 7$$

Given
$$x = \frac{4}{\sqrt{7} - \sqrt{3}}$$
, $y = \sqrt{7} - \sqrt{3}$,

prove that x and y are conjugates, then find the values of: $x^2 - 2xy + y^2$, $(x - y)^2$. What do you observe?

Solution

$$x = \frac{4}{\sqrt{7} - \sqrt{3}} \times \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} + \sqrt{3}} = \frac{4(\sqrt{7} + \sqrt{3})}{7 - 3} = \sqrt{7} + \sqrt{3}$$

$$y = \sqrt{7} - \sqrt{3} \therefore x, y \text{ (two conjugate numbers)}$$

$$x^2 - 2x y + y^2 = (\sqrt{7} + \sqrt{3})^2 - 2(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) + (\sqrt{7} - \sqrt{3})^2$$

$$= (7 + 2\sqrt{21} + 3) - 2(7 + 3) + (7 - 2\sqrt{21} + 3)$$

$$= 10 + 2\sqrt{21} - 8 + 10 - 2\sqrt{21}$$

$$= 12$$

$$(x - y)^2 = [(\sqrt{7} + \sqrt{3}) - (\sqrt{7} - \sqrt{3})]^2$$

$$= [\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}]^2 = (2\sqrt{3})^2$$

Remark:
$$x^2 - 2xy + y^2 = (x - y)^2$$

 $= 4 \times 3 = 12$



In the previous example, find the value of each of the following:

$$G = (x + y)(x - y)$$

$$x^2 - y^2$$

What do you observe?

Exercises (1-8)

Choose the correct answer:

$$\triangle \sqrt{50} - \sqrt{18} - \sqrt{2} =$$

$$(\sqrt{8} + \sqrt{2})^2 = \dots$$

Description The multiplicative inverse of
$$\frac{\sqrt{3}}{6}$$
 is $\frac{\sqrt{3}}{6}$ or $6\sqrt{3}$ or $2\sqrt{3}$ or $-2\sqrt{3}$)

E The next number in the pattern
$$\sqrt{3}$$
 $\sqrt{12}$ $\sqrt{27}$ $\sqrt{48}$ is $\sqrt{50}$ or $\sqrt{50}$ or $\sqrt{60}$ or $\sqrt{90}$ i

$$(\sqrt{30} \text{ or } \sqrt{2} \text{ or 2 or 2 } \sqrt{2})$$

$$(\sqrt{10} \text{ or } 10 \text{ or } 18 \text{ or } \sqrt{18})$$

$$-\frac{\sqrt{3}}{6}$$
 or $6\sqrt{3}$ or $2\sqrt{3}$ or $-2\sqrt{3}$)

- A If $X = 3 + \sqrt{2}$ then the conjugate is and the product of the number by its conjugate is ..
- B The multiplicative inverse for $(\sqrt{3} + \sqrt{2})$ in its simplest form is

• Think If
$$x^2 = 5$$
, then $(x + \sqrt{5})^2 = \dots$ or

Think If $\frac{1}{x} = \sqrt{s} - 2$, then the value of X in its simplest form is

$$E 3\sqrt{2} + \sqrt{8} - \sqrt{18} =$$

3 Simplify to the simplest form
$$2\sqrt{5} + 6\sqrt{\frac{1}{3}} - \sqrt{12} - 5\sqrt{\frac{1}{5}}$$

If
$$x = \frac{4}{\sqrt{7} - \sqrt{3}}$$
, $y = \frac{4}{\sqrt{7} - \sqrt{3}}$. Find the value of $x^2 y^2$

If
$$A = \sqrt{3} + \sqrt{2}$$
, $B = \frac{1}{\sqrt{3} + \sqrt{2}}$. Find the value of $A^2 - B^2$ in its simplest form.

(6) If
$$x = \sqrt{5} + \sqrt{2}$$
, $y = \sqrt{5} - \sqrt{2}$

Find the value of
$$\frac{x+y}{xy-1}$$
 in its simplest form

if
$$x = \sqrt{7} + \sqrt{5}$$
, $y = \frac{2}{x}$



Operations on the cube roots

Think and Discuss

You will learn how

To carry operations on the cube roots

key terms

6 Cube root.

For any two real numbers a, b .

$$\sqrt[4]{a} \times \sqrt[4]{b} = \sqrt[4]{a \times b}$$

For example: $\sqrt{5} \times \sqrt{2} = \sqrt{5} \times 2 = \sqrt{10}$

For example: $\sqrt{40} = (8 \times 5 = \sqrt{8} \times \sqrt{5} = 2\sqrt{5})$ $\sqrt[4]{-128} = \sqrt{-64 \times 2} = \sqrt{-64} \times \sqrt[4]{2} = -4\sqrt{2}$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
 where $b \neq a, a, b \in R$

 $\frac{\sqrt{12}}{\sqrt{3}} = \frac{1^3}{\sqrt{3}} = \sqrt{4}$ For example:



$$\sqrt[4]{\frac{a}{b}} = \sqrt[4]{\frac{a}{b}}$$
 where $b \neq 0$, $a, b \in R$

 $\sqrt{\frac{3}{2}} = \frac{\sqrt[4]{3}}{\sqrt{3}}$ For example :



Think: If we multiply both the numerator and the denominator by \$\forall 4\$, then find the product in its simplest form



Examples:

Simplify to the simplest form:

$$4\sqrt{54} + 8\sqrt[4]{\frac{-1}{4}} + 5\sqrt[3]{16}$$

Solution

Exercises (1-9)

Put each of the following in the form a \$\frac{1}{2} \text{b} where a and b are two integers, b is the least positive possible value:

Find the result for each of the following in the simplest form:

$$\xi = \frac{1}{2}\sqrt{56} - \sqrt{\frac{7}{2}}$$

Q
$$\sqrt{\frac{3}{4}} + \sqrt{\frac{2}{9}}$$
 E $\frac{1}{2}\sqrt{56} - \sqrt{\frac{7}{27}}$ **E** $\frac{1}{2}\sqrt{30} \times 6 \times 100$

- 3 If $a = \sqrt[4]{5} + 1$, $b = \sqrt[3]{5} + 1$ find the value of each of the following
 - A (a b)5

$$a + b)^3$$

Prove that:

B.
$$\sqrt[3]{54} \times \sqrt[3]{16} \div (\sqrt[3]{4} \times 6) = 1$$



Applications on the real numbers

Think and Discusi

You will learn how

To solve applications on square and cube roots.

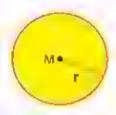
hey terms

- & Circle
- & Cuboid
- 9. Cube
- % Right circular cylinder
- **Sphere**

The circle:

Circumference of a circle = 2 π r length unit

area of a circle = r r2 square unit



where r is the length of the radius in a circle, π is the (approximate ratio).



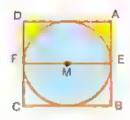
Examples

Find the circumference of a circle whose area is $38.5 \text{ cm}^2 \ (\pi = \frac{22}{2})$

Solution

The area of the circle = π r²

38.5 =
$$\frac{22}{7}$$
 r² \therefore r² = $\frac{38.5 \times 7}{22}$ = $\frac{49}{4}$
 \therefore r = $\sqrt{\frac{49}{4}}$ = $\frac{7}{2}$ = 3.5 cm



In the opposite figure, the circle M is inside the square ABCD If the area of the yellow sector is $10^{-\frac{5}{7}}$ cm², find the perimeter of the sector($\pi = \frac{22}{7}$)

Solution

We suppose that the length of the raidus in a Circle = r

.. The side length of the square = 2r

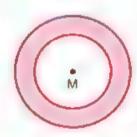
The area of the yellow color = the area of the rectangle AEFD - the area of semi circle

$$\therefore r^2 = 25 \quad \therefore r = 6 \text{ cm}$$

The perimeter of the yellow sectors = (AE + AD + DF) + $\frac{1}{2}$ the circumference of the circle = (5 + 10 + 5) + $\frac{1}{2}$ × 2 × $\frac{22}{2}$ × 5 = 35 $\frac{5}{2}$ cm



- A circle whose area is 64 n cm^2 . Find the length of its radius, then find its circumference approximating it to the nearest integer ($\pi = 3.14$).
- In the figure opposite: AB is the diameter of a semi circle. If the area of that region is 12.32cm². Find the circumference of that figure.
- In the opposite figure: there are two circles have the same center "concentric" of center M. If the lengths of their radii are 3cm and 5cm. Find the area and the circumference of the colored region in the terms of π.



The cuboid

It is a body whose six faces are of a rectangular shape such that every two opposite faces are congruent:

If the lengths of its edges were x, y, z, then:

The lateral area = the perimeter of the base × the height



The lateral area = the lateral area + 2 x the area of the base

The total area =
$$2 (xy + yz + Xz)$$
 square unit

The volume of the cuboid = the area of the base x the height

The volume of the cuboid = $x \times y \times z$ cubic unit

A special case: the cube

It is a cuboid whose edges are equal in length. If the length of one edge = L length unit, then:

The area of each face = L2 square unit

The lateral area of each face = 4L2 square unit

The total area = 6L2 square unit, the volume of the cube = L3 cubic unit





Examples



Harman ex

Find the total area of a cube whose volume is 125 cm³

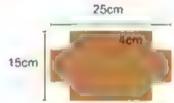
Solution

The volume of the cube = L^3 \therefore 125 = L^3 \therefore $L = \sqrt[3]{125} = 5$ cm The total area = $6L^2 = 6 \times (5)^2 = 150$ cm²



Practice

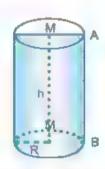
- Find the total area of a cuboid whose volume is 720cm3 and height 5cm with a squared shape base.
- Which is more in volume: A cube of 294 cm² area or a cuboid with the following dimensions: $7\sqrt{2}$, $5\sqrt{2}$, 5 cm.
- A rectangular hard piece of paper has a length of 25 cm and a width of 15 cm. A square whose side = 4 cm was cut from each of its four corners. Then, the projected parts were folded to form a shape of a cuboid. Find the volume and the total area of that cuboid.



The right circular cylinder:

It is a body that has two parallel congruent bases each is a circular shaped surface, while its lateral surface is a curved surface called cylindrical surface

If M, M' are the bases of the cylinder, then M M' is the height of cylinder.





Let's think If A ∈ the circle M, B ∈ the circle M',

) Then, if we cut the lateral cylindrical surface at AB and we stretch That surface, we get the surface of the rectangle A B B, V,

Then, AB = height of cylinder, A A' = the perimeter of the base of the cylinder.



The area of the rectangle A B B A = the lateral area of the cylinder

The lateral area of the cylinder = the perimeter of the base \times height = 2π r h (square unit) the total area of the cylinder = area of lateral surface + sum of the areas of the two bases

$$= 2 \pi r h + 2 \pi r^2$$
 (square unit)

the volume of the cylinder = base area \times height = π r² h (cubic unit)



Example

A piece of paper has shape of a rectangle ABCD in which AB = 10cm, BC = 44cm. It was folded to form a right circular cylinder such that AB is congruent to DC. Find the volume of the resulted cylinder. ($\pi = \frac{22}{3}$).

Solution

The perimeter of the cylinder base = 44 cm.

$$2\pi r = 44$$

$$2 \times \frac{22}{7}$$
 r = 44

$$\therefore$$
 r = 7cm

The volume of the cylinder $= \pi r^2 h$

$$=\frac{22}{7}\times (7)^2\times 10$$





Practice

Find the volume and the total area of a right circular cylinder in which the length of base radius = 14 cm and the height is 20 cm.

- Find the total area of a right circular cylinder of volume 7536 cm³ and height 24 cm (π = 3.14)
- Which is more in volume: a right circular cylinder of radius 7cm and height 10 cm or a cube whose edge length is equal to 11cm

The sphere:

It is a body of curved surface in which the points have the same distance (r) from a constant point inside it (the center of the sphere)...

If the sphere is cut by a plane passing by its center, then the resulted section is a circle whose center is the center of a sphere where its radius is the radius of a sphere (r).

Volume of the sphere =
$$\frac{4}{3} \pi r^3$$
 cubic units.
area of the sphere = $4 \pi r^2$ square units.





The volume of the sphere is $562.5~\pi$ cm 3 . Find its surface area.

Solution

the volume of sphere =
$$\frac{4}{3} \pi r^3$$

 $562.5 \text{ T} = \frac{4}{3} \times \pi r^3$
 $\therefore r^3 = 562.5 \times \frac{3}{4} = 421.875$
 $r = \sqrt[3]{421.875} = 7.5 \text{ cm}$

the surface area of sphere = 4 π r² = 4 \times π (7.5)² = 225 π cm²



Find the volume and the surface area of a sphere whose diameter is 4.2cm ($\mathcal{I} = \frac{22}{7}$)

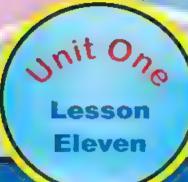
Exercises (1-10)

Choose the correct answer

- The lateral area of a right circular cylinder whose base diameter length b and height h is $(\pi b^2 h, \pi bh, 2\pi bh, \pi bh^2)$.
- The volume of a sphere whose diameter length is $6 \text{cm} = \text{cm}^3$ (288, 12 π , 36 π , 288 π)
- The edge length of a cube whose volume is $2\sqrt{2}$ cm³ = ___ cm. $(\sqrt{2}, 2, 8, 1.5)$
- The radius length of a right circular cylinder whose volume is 40π cm³ and height 10 cm = cm (5, 3, 2, 1).
- The volume of a cuboid whose dimensions are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ cm is (6, 36, $6\sqrt{6}$, $18\sqrt{2}$).

Complete

- A The radius length of a sphere whose volume is $\frac{9}{2}\pi$ cm³ is cm.
- A right circular cylinder whose base radius length r and height h, then its lateral area is and its volume is
- The total area of a cube of edge length = 4 cm is cm².
- D. The lateral area of the cuboid = .
- A sphere with volume 36 π cm³ is placed inside a cube, if the sphere touches the cube's six faces. Find
 - the radius of the sphere.
- & the volume of the cube.
- A metal sphere with diameter 6 cm has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height.
- Find the height of a right circular cylinder whose height is equal to its base radius and its volume is $72 \pi \text{ cm}^3$.
- A hollow metal sphere with internal radius 2.1 cm and external radius 3.5 cm. Find its mass to the nearest gram given that the mass of a cubic centimeter of such a metal is 20 gm ($\pi \frac{22}{3}$)



Solving Equations and Inequalities of first degree in one variable in R

Think and Discuss

You will learn how

- To solve equation of first degree in one variable in R
- To solve inequalities of first degree in one variable

key termi

- 🦠 -equation
- degree of an equation
- 5 Inequality
- degree of an inequality
- Solution of an equation
- Solution of an inequality

First-Solving Equations of first degree in one variable in R

We know that: The equation 3X - 2 = 4 is called an equation of first degree where the exponent of the (unknown) variable X is 1. To solve that equation in R

 $3 \times -2 = 4$ By adding 2 to the sides of the equation

3 x = 6 (we can multiply by the multiplicative inverse of the coefficient of X)

$$\frac{1}{3} \times 3x = \frac{1}{3} \times 6$$

$$x = 2$$

i.e the solution set { 2 }

This solution can be graphed on the number line as shown in the figure opposite.



Examples



Find the solution set of the equation y + x + 1 = 2, in R, then graph the solution on the number line.

Solution

$$\sqrt{3} \times -1 = 2$$
 $\sqrt{3} \times = 3$
 $\sqrt{3} \times \sqrt{3} \times = \sqrt{3} \in \mathbb{R}$

The solution set is $\{\sqrt{3}\}$



This solution can be graphed on the number line as shown in the figure opposite.

Find The solution set for the equation $x + \sqrt{2} = 1$, in R, then graph the solution on the number line.

Solution

$$x + \sqrt{2} = 1$$

$$\mathbf{x} + \sqrt{2} = 1$$
 $\therefore \mathbf{x} = 1 - \sqrt{2} \in \mathbf{R}$



This can be graphed on the number as shown in the figure opposite.

Practice

Find the solution set for the following equations in R, then graph the solution on the line number.

$$A 5x + 6 = 1$$

$$B 2x + 4 = 3$$

$$C = 2x - 3 = 4$$

$$0 \times +5 = 0$$

Second . Solving inequalities of the first degree in one variable in R, graphing the solution on the number line.

The following properties are used to solve the inequality in R. The solution set is written in the form of an interval

If A . B. C were real number where A< B, then:

addition property.

If C > 0 then A × C < B × C. property of multiplication by a positive real number

If C < 0 then A × C > B × C. property of multiplication by negative real number



Examples



Find the solution set for the inequality $2 \times -1 \ge 5$ in R and represent the solution set graphically.

Solution

By adding 1 to the sides of the inequality it becomes $2 \times 2 = 6$ by multiplying the side of the in equality by $(\frac{1}{2} > 0)$ $x \ge 3$

.. The solution set in R is [3, ∞[and it is graphed by green color ray on the number line.

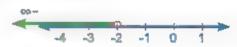


Find the solution set for the inequality 5 - 3 x > 11, in R, then represent the solution graphically.

Solution

By adding (-5) to the sides of the inequality then -3 x > 6 by multiplying the sides of the inequality by $(-\frac{1}{3} < 0)$ we get :

: x < -2



i.e., the solution set in R is est]- 00 , -2[

and it is represented by the green color ray on the number line.

Find the solution set for the in equality -3 < 2x -1 < 5 in R and represent the solution graphically.

Solution

by adding (1) to the sides of the inequality-3 + 1 < 2x - 1 + 1 < 5 + 1

Namely, $-2 \le 2 \times \le 6$, and by multiplying the sides of the inequality by $(\frac{1}{2} \ge 0) -1 \le x \le 3$

.. the solution set in R is [-1, 3] and it is graphed on the number line by the green color.



Exercises (1-11)

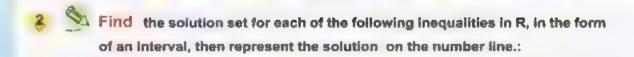
- Complete the following to have a true sentence where X ∈ R
 - A If 5 x < 15, then x</p>
 - B If x -3 > 4, then x ...
 - C If -2 x < 3, then x
 - If 1 x > 4, then x

- Find The solution set for each of the following inequalities in R in the form of intervals, then graph the solution on the number line.:
 - A 3x -1 < 5
- B 2x+5≥3
- C 2x+3<1
- D = 5 x > 3
- E 1 5x < 6
- € 1 x+1 < 2
- Find the solution set for each of the following inequalities in R in the form of an interval, then represent the solution on the number line.
 - $A = 1 \le 2x + 1 \le 5$
- B -5<2x-3<1
- C -3 < 4x 7 < 5
- D 4<3x+4<7
- E 1 < 5 x < 3
- E 1<3-2x<5
- Find the solution set for each of the following inequalities in R, write it in the form of an interval, then represent the solution on the number line:
 - $A = 3 \le -x < 3$

- B -3 <2x-1<5
- © $\sqrt[3]{-8}$ ≤ x + 1 ≤ $\sqrt{9}$ D 5 < 3 x ≤ 3²

General Exercises

- Complete the following to have a true sentence:
 - $A \sqrt{9} + \sqrt[3]{-8} = .$
 - B. A vessel that has the shape of a cub whose capacity is 8 liters, the length of the edge of its interior face is cm.
 - The solution set for the equation $x^2 + 9 = 0$ in R is
 - D. $(\sqrt{3} + \sqrt{2})^2 + (\sqrt{3} \sqrt{2})^2 =$
 - A rectangle of dimensions are $(\sqrt{5} + 1)$, $(\sqrt{5} 1)$ cm has an area ofcm2.
 - E 354 8-16 = 3
 - **G** [-1, 5]]-1, 5[=
 - Herefore The solution set for the equation $\sqrt{2} \times 1=3$ in R is
 - A sphere whose diameter is equal to 6 L (length unit) has a volume of (cubic unit).
 - J | √-125 | = √



Market Market Market

$$E 4x \le 5x + 2 < 4x + 3$$

If
$$x = \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - \sqrt{5}}$$
 prove that $x + \frac{1}{x} = 22$

Find in the simplest form:
$$\sqrt{54} + 4\sqrt{\frac{1}{4}} = \sqrt{-2}$$

If
$$x = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$$
, $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$ Find the value

$$\triangle x^2 + y^2$$

and prove that $x^2 + y^2 = 38 \times y$

If
$$X = \sqrt[3]{5} + 2$$
, $y = \sqrt[3]{5} - 2$ Find the value of $(x + y)^3 + (x - y)^3$.

9 If
$$x = \sqrt{5} = \sqrt{3}$$
, $y = \frac{2}{\sqrt{5} = \sqrt{3}}$. Find the value of $(X^2 + 2xy + y^2)$

49 If
$$A = \sqrt{3} + \sqrt{2}$$
, $B = \sqrt{3} - \sqrt{2}$,

find the value of $(A^2 - AB + B^2)$

11 If
$$x = \frac{3\sqrt{5} + 5\sqrt{2}}{\sqrt{5}}$$
, $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

Prove that
$$\frac{x^2 + y^2}{xy} = 38$$

rechnology

Find: $\sqrt[3]{27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0,125}$ Open microsoft office Excel and record the shown numbers in cells A1, D1 and B1.



To Find the cube root of cell A1, write the formula A1^(1/3) in the cell F1, then Enter... the result is 3.

To find the square root of cell B1, write the formula B1^(1/2) in the cell H2, then Enter... the result is 3.5.

To find the cube root of cell J1, write the formula D1^(1/3) in the cell J2, then Enter... the result is 0.5.

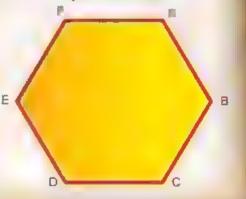
Write down the sum of F2+H2+J2 in cell L2, after you do a click on = then the sum is 1.

Activity



Activity Draw a regular hexagon ABCDEF whose side is equal to 4 cm.

- Find the measure of its interior angle
- 2 Draw its diagonals AD , BE , CF then deduce the length of each diagonal without measuring them by the ruler.
- draw a circle that passes by its vertices.
- Find its area.







- A]-3, 2] n R = .
- B. The multiplicative inverse for $-\frac{\sqrt{2}}{6}$ is
- $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, Complete in the same pattern.
- Q If $x = \sqrt[4]{3} + 7$, $y = \sqrt[4]{3} 7$ then $(x + y)^3 = \cdots$
- E The circle of 20X cm circumference has an area of π cm²

A a cube of 64 cm³ has a lateral area of =...cm² (4 or 8 or 64 or 96)

B.
$$\sqrt{12} - \sqrt{3} =$$

(3 or
$$\sqrt{3}$$
 or $2\sqrt{3}$ or $3\sqrt{3}$)

C The multiplicative inverse for $\frac{\sqrt{6}}{12}$ is $(\frac{12}{\sqrt{6}} \text{ or } \sqrt{\frac{6}{12}} \text{ or } -2\sqrt{6} \text{ or } 2\sqrt{6})$

$$(\sqrt[4]{52} \text{ or } \sqrt[4]{2} \text{ or } 2\sqrt[4]{2} \text{ or } 4\sqrt[4]{2})$$

$$[-3, 4] - \{-3, 5\} =$$

- 3 Reduce to the simplest from $2\sqrt{18} + \sqrt{50} + \frac{1}{3}\sqrt{162}$
- a lead cuboid in which the lengths of its faces are 77cm, 24cm and 21cm. It was melted to form a sphere. Find the radius of that sphere. ($\pi = \frac{22}{7}$)

5 If A =
$$\frac{4}{\sqrt{7} - \sqrt{3}}$$
, B = $\frac{4}{\sqrt{7} + \sqrt{3}}$ find the value of $\frac{A-B}{AB}$

- Using the number line find]-1, 3] ∪ [0, 5[in the form of an interval
- Find the lateral area for a right circular cylinder of volume 924cm³ and height 6cm ($\pi = \frac{22}{2}$).
- If $x = \sqrt{10} + 2$, $y = \sqrt[3]{26} 1$. Give an estimate for the product of X × Y and use the calculator to find the difference between your estimate and the exact answer.
- Find the Solution set in R and graph the solution on the number line.

B
$$x + 2\sqrt{3} = 3$$





Linear Relation of two variables

Think and Discuss

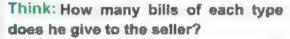
You will learn how

- Relations of two variables
- To graph the linear relations of two variables

key terms

- Variable
- Relation
- & Linear equation

A person has some bills of LE 50 and LE 20. He bought an electrical apparatus for LE 390.



Suppose: x represents the number of

fifties bills, then the value of what he has of these bills is L E 50x, y represents the number of Twenties bills, then the value of what he has of these bills is L.E 20y

Required is to know: x and y that verify the equation:

$$50 \times + 20 \text{ y} = 390$$

This relation represents a linear equation in two variables. Dividing both sides over 10 produces the following equivalent equation:

$$5x + 2y = 39$$

 $y = \frac{39 - 5x}{2}$

Note the x and y are natural numbers. Therefore, x should be an odd number.

The following table can be created to know the different

possibilities of giving bills to the				
seller: a bill of L E50 and 17 bills				
of L.E 20, or 3 bills of				
LE 50 and 12 bills of L.E 20,				
or 5 bills of L.E 50 and 7 bills				
of L.E 20, or 7 bills of 50 and 2				
bills of L.E 20.				

Х	У	(x , y)
1	17	(1,17)
3	12	(3, 12)
5	7	(5,7)
7	2	(7,2)
9	negative	refused

Practice

- A person has some bills of L E 5 and some of L.E20. He bought some goods from
- amount in the two types of bills which he has? The perimeter of an isosceles triangle is 19cm. What are the different possible lengths of its sides? Side length ∈ Z+

a shopping center for L.E75. What are the different possibilities of paying this

Remember The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

The Relation of two variables

$$a \times by = c \text{ where } a \neq 0$$
, $b \neq 0$ is called a linear relation of two variable x

and y and can be described by a set of ordered pairs (x, y) verifying this relation.

Example:

Refer to the relation 2x = v = 1

If
$$x = 1$$
, $\therefore y = 1$ $\therefore (1, 1)$

If
$$x = 0$$
, $x = -1$ $x = 0$, $x = -1$

If
$$x = 3$$
, $x = y = 5$ $x = (3, 5)$

If
$$x = -1$$
, $\therefore y = -3$ $\therefore (-1, -3)$

satisfies the relation

satisfies the relation

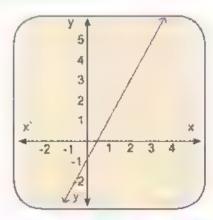
satisfies the relation

satisfies the relation

Thus, there are an infinite number of ordered pairs satisfying the relation

Note that:

- The linear relation 2x -y = 1, can be represented graphically by using any of the ordered pairs obtained before.
- Each point ∈ the straight line (in red) is represented by an ordered pair whose elements satisfy the linear relation 2x - y = 1.





Electronic terror

Find four ordered pairs satisfy each linear relation and represent it graphically:

$$\Phi x + y = 3$$

$$-2y = 5$$

$$\Psi x = 1$$

- 2. Find the value of b, where (-3, 2) satisfies the relation 3x + b y =1.
- 3 Find the value of k, where (k, 2k) satisfies the relation x + y = 15.

Graphing the Relation of two Variables

The relation

ax + by = c

where a and b or both are not equal zero.

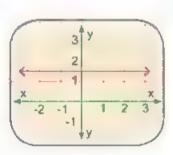
is called a linear relation of two variables x and y and can be represented graphically by a straight line.

for
$$a = 0$$

The relation is represented by a straight line parallel to x-axis.

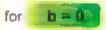
Example: 2y = 3

i.e.: $y = \frac{3}{2}$ is represented by the red line which passes through the point $(0, \frac{3}{2})$ and is parallel to x-axis.



Special case:

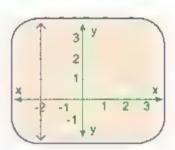
the relation y = 0 represents the x-axis



The relation is represented by a straight line parallel to y-axis.

Example: x = -2

is represented by the red line which passes through the point (-2, 0) and is parallel to y-axis.



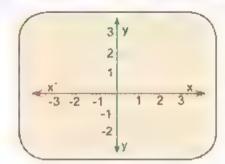
Special case:

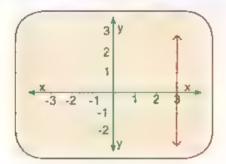
the relation x = 0 represents the y-axis.



Graph each relation of the following:

Find the relation that is represented by the red line in each figure below:







Example:

Graph the relation: x + 2y = 3

Solution

Choose some ordered pairs that satisfy the relation:

For
$$y = 2$$

$$x = 1$$
 (-1, 2)

$$(-1, 2)$$

$$y = 0$$

v = -1

$$A \cdot X = 3$$

$$x = 3$$
 (3,0)

$$\therefore x = 5 \qquad (5, -1)$$

satisfies the relation satisfies the relation satisfies the relation and so on

The following table lists these data:

x -1 3 5 0
y 2 0 -1
$$\frac{3}{2}$$

The red line represents this relation.

Discuss with your teacher:

- What happens to the value of y when increasing the value of x?
- When does the line representing the relation ax + by = c pass through the origin 0?

Exercises (2-1)

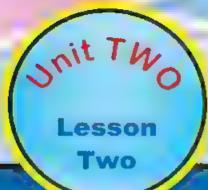
Graph each of the following:

$$x + y = 2$$

$$2 x - y = 3$$

Graph the relation 2x + 3y = 6. If the straight line representing this relation intersects the x-axis in point A and the y-axis in point B. Find the area of the triangle OAB, where 0 is the origin.





The Slope of a line and real-life Applications

Hink and Discuss

You will learn how

- The slope of a line
- Real-life applications on the slope of a line

key termi

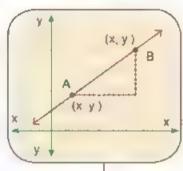
- % Slope.
- Positive slope.
- Negative slope.
- Zero-slope.
- Undefined slope.

When observing the motion of a point on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2) , where

x, > x, and A, B ∈ line, then:

- the change in x-coordinate

 = x₂ x₁, and is called the horizontal change.
- 2 the change in y-coordinate = y₂ y₁ is called the vertical change and may be positive, negative or zero.

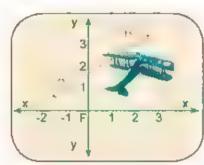


In the following examples you will learn different cases of the vertical change $(y_2 - y_1)$:



Example (1)

If: A (-1, 1) and B (2, 3). then: the slope of \overrightarrow{AB} = $\frac{3-1}{2-(-1)} = \frac{2}{3}$



Note that:

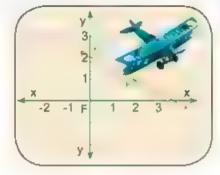
- The point A moves on the line upwards to the point B
- The slope of the line is positive.



Example (2):

If: A (0, 2), B (2, 1);

then: the slope of $\overrightarrow{AB} = \frac{1-2}{2-0} = -\frac{1}{2}$



Not that

The point A moves on the line downwards to the point B

2 y₂ < y₁ 3 The slope of the line is negative

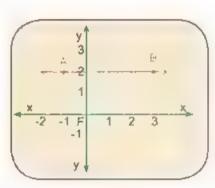


Example (3):

If: A (-1, 2) and B (3, 2).

then: the slope of the line

$$AB = \frac{2 - 2}{3 - (-1)} = \frac{0}{4} = 0$$



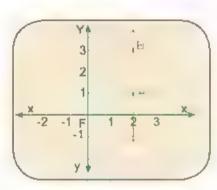
•Not that:

- The point A moves horizontally to point B.
- $y_2 = y_1$
- The slope of the line = zero



Example (4):

If: A = (2, 1) and B(2, 3) then; we can not calculate the slope. Because the definition of the slope is conditioned to have a change in the x-coordinate i.e. $x_2 - x_1 \neq 0$



Not that

- The point A moves vertically to point B.
- $2 x_2 = x_1$
- The slope of the line is an underlined number.



Practice:

- Find the slope of the straight line AB in each of the following cases:
- Find the slope of AB, BC and AC, where A(2, -1), B(3, 2), and C(4, 5) and represent each line graphically. What do you observe?
- Choose the true answer:

First: The following table shows the relation between x and y as follows:

(y = x + 4 or y = x + 1 or y = 2x - 1 or y = 3x - 2)

Second: If (2, -5) satisfies the relation 3x - y + c = 0, then c = (1, -1, -11, -11)

Third: (3, 2) does not satisfy the relation (y + x = 5, 3y - x = 3, y + x = 7, y - x = 1)

Fourth: An irrigation machine consumes 2 47Litres of diesels to work for 3 hours. If the machine works for 10 hours, it consumes... litres. (7.2, 8, 8.4, 9.6)

Find the slope of the line AB, where A(-1, 3) and B (2, 5). Is the point c (8, 1) ∈ AB?

Real-life Applications on the slope of a line.

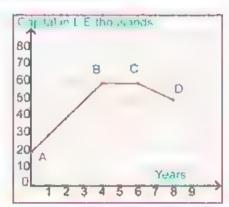
Application (i):

The opposite figure shows capital change of a company during 8 years.

- A Find the slope of AB . BC and CO What is the meaning of each?
- Find the starting capital of the company.

Solution

A(0, 20), B(4, 60), C(6, 60), D(8, 50)



First: The slope of = AB = $\frac{60 - 20}{4 - 0} = 10$, shows the acreasing of the capital during the list or years with a rate of 10 thousand pound

The slope of
$$\frac{1}{BC} = \frac{60 + 60}{6 - 4} = 0$$
,

means that the capital was constant during the fifth and sixth years.

The slope of
$$\overrightarrow{CD} = \frac{50-60}{8\cdot6} = -5$$

shows the vecreasing of therap taildining the last two years with a rate of 5 Thousand pound.

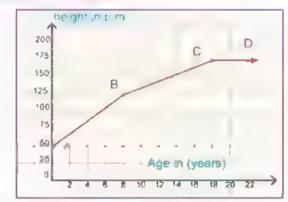
Second: Starting Capital # the y-coordinate of the point A = LE 20 000



Practice:

The opposite figure shows the relation between the height of a person (in cm) and his age (in years).

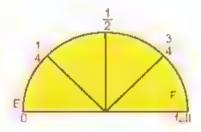
First: Find the slope of AB , BC and CD What is the meaning of each?



Second:Calculate the difference between the height of this person as he was 8 years old and his height as he was 30 years old.

Application (2):

Hazem filled up the 40 Litres tank of his car As covering a distance of 120 km, the fuel gage shows the rest of fuel is $\frac{3}{4}$ of the tank. Draw a diagram to show the relation between the amount of fuel in the tank and coverd distance (This relation is linear). Calculate the coverd distance as the tank is totally getting empty.



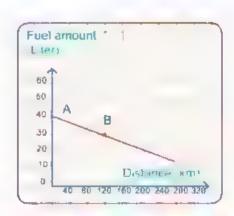
Solution

On the starting point: A (0, 40)

After covering 120 km B = (120, 30)

The slope of AB =
$$\frac{30-40}{120-9} = \frac{.1}{12}$$

This slope means the fuel amount decreases with a rate of 1L per 12 km, which means 1L is enough to cover a distance of 12 km.





The coverd distance that make the tank empty =
$$\frac{\text{Fuel Amount}}{\text{Decreasing Rate}} = \frac{40}{\frac{1}{12}}$$
$$= 40 \times \frac{12}{1} = 480 \text{km}.$$

Note that AB intersects the distance-axis in the point (480, 0) which gives the required distance.

Exercises (2 - 2)

Complete to make a true statement:

THE REAL PROPERTY.

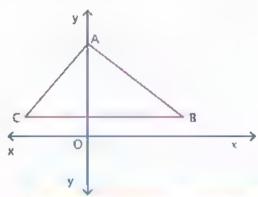
- Let A (1, 3) and B (2, 1). Then the slope of AB =
- # If (-1, 5) satisties the relation, 3x + ky = 7, then k =
- The slope of any line parallel to x-axis =
- The slope of any line parallel to y axis =
- E If A, B and C are collinear points. Then the slope of AB = the slope of
- Essam has 10 bills of LE 5 and other bills of LE 20. He bought some goods from a shopping center for LE 65. Determine the different possibilities to pay this amount of money. Find the relation and graph it.
- The selling price of a computer table is LE 100 and its chair is LE 50. If the store sells in one week with LE 500. What are the represented expectations to the number of computer tables and chairs?

Represent the relation graphically.

In the opposite figure ABC is a triangle. Complete by using one of the following words:

(positive, negative, zero, undefined)

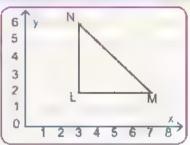




Unit 2: Lesson 2

In the opposite figure:

LMN is a right angled triangle at L, where m (\angle M) = 45°, given that L (3, 2) and M (7, 2). Find the coordinates of N and calculate the slope of MN.

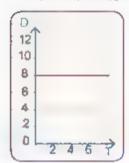


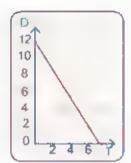
The following diagrams shows the relation

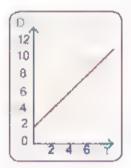
1 2 3 4 5 6 7

between the coverd distance (in m) and the elapsed time (in sec) of an object.

Determine the position of the object at the starting motion and its position after 6 seconds when t = 6 sec. Find the slope of the line in each case, and state what it represents.



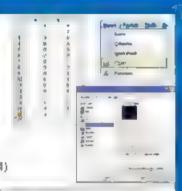




Technology:

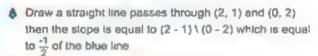
open micro soft office Excel to draw the two axes x-y then write the shown number as in fig(1) in the first column A and column B.

Do a mouse click to shade the two columns then from the menue INSERT choose CHART as in fig (2) then XY SCATTER as in fig (3) then press NEXT and FINISH, the x-y axes appear



Do a mouse click on ____ from the menue of drawing downward of the page EXCEL and determine the values of dots as shown in fig (4)

Do a mouse click on 📆

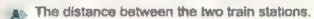




- Oraw a straight line passes through (2, 2) and (-2, 2), then the slope is equal to (2 - 2)\(-2 - 2\) which is equal to zero i.e. the line is parallel to the x axis- the yellow line
- © Draw a straight line passes through (2, -1) and (2, 5), then the slope is equal to (5-(-1)) \ (2-2), then the slope is undefined, i.e. the line is parallel to the y-axis the red line.

Activity

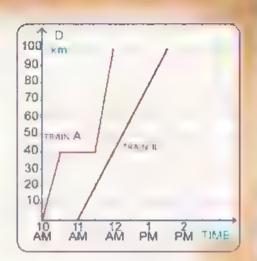
The opposite diagram shows the relation between the coverd distance (in km) and elapsed time (in h) of two trains A and B over the distance between two train stations. Use the diagram to find:



is The elapsed time of each train.

the second to the second to the second

- The average speed of each train.
- The meaning of the horizontal segment in the diagram of train A.



The coverd distance

The averege speed = Total elapsed time needed to coverd this distance



Unit Test



Choose the true answer:

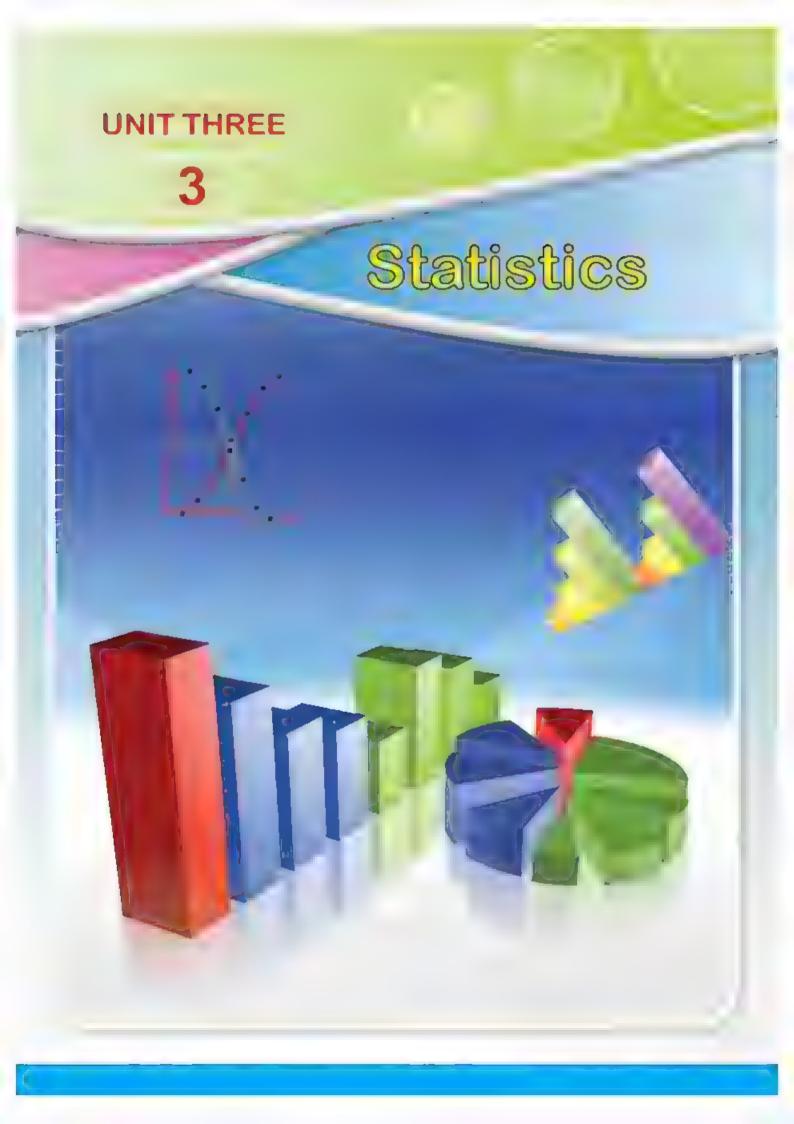
- Which of the following ordered pairs satisfies the relation $2 \times + y = 5$ ((-1, 3), (1, 3), (3, 1), (2, 2))
- Which of the following relations is the relation illustrated in the table.

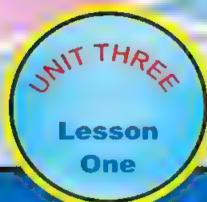
$$(y = x + 7, y = x - 7, y = 3x + 1, y = x + 1)$$

- Let A (3, 5) and B (5, -1), then the slope of $\overrightarrow{AB} = (-\frac{1}{3}, -3, 3, \frac{1}{3})$
- The line represented by the relation 3x + 8y = 24, intersects the y-axis at the point ((0, 8), (8, 0), (0, 3), (3, 0))
- 2 Let A (2, -1), B (10, 3) and C (2, 3) Find the slope of AB BC and AC

Draw the triangle ABC on a square grid, then tell the type of the trinagle according to its angles.

Atef filled up the 50 L-tank of his car As coverd a distance of 100km, the fuel gage shows the rest of fuel is \(^4_5\) of the tank. Draw a diagram to show the relation between the amount of fuel remaining in the tank and the distance coverd. What is the coverd distance as the tank is getting totally empty?





Collecting and Organizing data

Think and Discuss

You will learn how

- To collect and organize data
- Using frequency tables with sets

key terms

- Collecting data
- Crganizing data
- Frequency table with sets

If you study the traffic jam problem and its possible solutions:

- What are the sources of your data?
- How can you collect data about such a problem?
- What are the statistical methods you will use to analyze the data?



What do you suggest to solve that problem and improve traffic fluidity?

Collecting data

Let's work-together Cooperate with your classmates on collecting data from their sources through distribution of roles.

- Group 1: Collects primary data about the problem under discussion through a survey that asks about (the means of transportation - Roads conditions - time of traffic jam -Existence of traffic signs - existence of security).
- B Group 2: Collects secondary data about the problem under discussion from the traffic reports - the internet - the mass media).
 - Group 3: Observes the crowdest roads, the drivers' behavior and their obedience to traffic rules the pedestrians' commitment to the virtues to the road as well as crossing the roads at safe places.

Organizing and Analyzing data

Cooperate with your classmates on making afrequancy table that represents the means of transportation used by your classmates..

Means of transportation

Subway bus Private Taxi bicycle on foot total

Frequency

Determine the most used means of transportation (The mode)

- Is that means suitable? does it help solving the traffic jam problem? why?
- What do you suggest to solve this problem according to the results you have collected?

Organizing data and representing them in frequency tables



Below are the scores of 30 students in an examination

1	7	10	7	4	5	8	6	7	13	12 9 17
ı	2	9	11	12	11	9	15	12	13	9
1	5	14	19	3	9	14	3	13	8	17

Required: forming a frequency table with sets that represents that data

Solution

To form a frequency table with sets, follow the following steps:

First find the highest and the lowest values of the collected data?

let the previous collected data be X

then: $X = \{x : 2G \times G \ 19\}$

f.e: X values begins with 2 and ends in 19

i.e: the range = the highest value - the lowest value = 19 - 2 = 17

Second: divide set X into a number of separate subsets each of them is equal in range let them be 6 sets. A The range of the set = \frac{17}{6} it e approximated to 3

Third: the subsets are as follow.

The first set	2 -
The second set	5 -

the third set	8 -	
The Fourth set	11	and so on

2- means the set of data greater than or equal to 2 and less than 5 and so on.

Fourth: Record the data in the following table:

Set	tally	frequency
2-	1111	4
5 -	MI 1	6
8 -	111/11	7
11 -	111/11/1	8
14 -	///	3
17	//	2
Total		30

Fifth: Delete the tally column from the table to get the frequency table with sets. It can be written either vertically or horizontally. The following is the horizontal form of the table:

Sets	2 -	5	8 -	11	14 -	17 -	total
Frequency	4	6	7	8	3	2	30

Below are the weekly wages of 40 workers in a factory in L.E.

47	71	36	94	54	64	87	89	62	57
64	61	AA	62	70	86	EG	22	60	76
31	91	-4-4	32	10	00	30	32	09	30
70	40		00	0.5	-00	00	07	00	
(A	48		90	65	99	90	0/	υU	22
95	75	81	84	7R	38	49	94	48	59
					~~		~ .		

Required. Form a frequency table with sets (use the subsets. 30-, 40-, 50-,90-). What is the set with the highest frequency? what is the set with lowest frequency?

The following are the scores of 30 students in a monthly math exam.

	25	35	40	20	30	37	40	33	22	38 37 31
	35	36	28	37	39	28	32	26	29	37
١	23	34	35	36	29	38	40	35	37	31

Required:

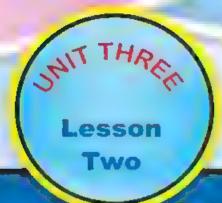
Form a frequency table with sets for these scores.

By Find the total number of excellent students. The excellence rate is 36 marks or more.

3 The following table shows the days-off which 40 workers got during a year.

Required

- Form a frequency table with sets to represent the data above
- Find the number of workers who got more than 20 days-off a year long.



The Ascending and Descending Cumulative Frequency Table and Their Graphical Representation

Think and Discuss

You will lears how

- To Form both ascending and descending cumulative frequency tables
- To represent both ascending and descending cumulative frequency tables graphically

key term/

- The frequency distribution.
- 5 The frequency table.
- The ascending cumulative frequency table
- The descending cumulative frequency table
- The ascending cumulative frequency curve
- The descending cumulative frequency curve

First: Ascending cumulative frequency table and its graphical representation.



Examples

The following table shows the frequency distribution for the heights of 100 students in a school in centimeters

Tail (sets) in c.m	115-	120-	125-	130-	135-	140-	145-	Total
Number of students (frequency)	8	12	19	23	18	13	7	100

- How many students are with height less than 115cm?
- 2 How many students are with height less than 135cm?
- 3 How many students are with height less than 145cm?

Form the ascending cumulative frequency table for these data and represent them graphically.

Solution,

- Are there students with height less than 115c.m? No
- Are there students with height less than 135c.m? How many?
 yes, 62 student.
- How can you calculate the number of students with height less than 145 cm? Add the number of students in the sets of height less than the set 145.

Now, to answer the previous questions in an easier way, form an ascending cumulative frequency table as follows.

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	0
Less than 120	(f) + 8 = (8)
Less than 125	® + 12 = 20
Less than 130	20 + 19 = 39
Less than 135	39 + 23 = 62
Less than 140	@ + 18 = 80
Less than 145	804 13 = 93
Less than 150	3 + 7 = 100

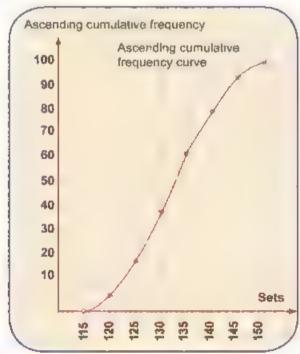
Cassanding consists	# frequency table
Upper boundaries of sets	Ascending cumulative frequency
Less than 115	zero
Less than 120	8
Less than 125	20
Less than 130	39
Less than 135	62
Less than 140	80
Less than 145	93
Less than 150	100

To represent the ascending cumulative frequency table graphically:

Specify the horizontal axis to the sets and the vertical axis to the ascending cumulative frequency

i.e.

- 2 Choose a drawing scale to draw the vertical axis such that the ascending cumulative frequency axis can hold the number of elements in a set
- 3 Represent the ascending cumulative frequency for each set and draw its line graph successively.



Second: The descending cumulative frequency table and its graphical representation.:

Of the previous frequency distribution which shows the heights of 100 students in a school in centimeters.

Find: The number of students with heights of 150cm and more..

The number of students with heights of 140cm and more..

The number of students with heights of 125cm and more...

Form the descending cumulative frequency table and represent it graphically

Solution

There are no students with heights of 150cm and more.

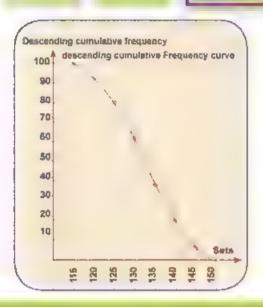
The number of students with heights of 140cm and more is 7 + 13 = 20 students. The number of students with heights of 125cm and more is

To answer these questions in an easier way, form the descending cumulative frequency table as follows:

A W/A FA-	Ascending	of sets cumulative frequency
Lower limits of sets	cumulative frequency	115 and more 92 + 8 = 100
115 and more	100	120 and more (6) + 12 = (92)
120 and more	92	125 and more 61 + 19 = 80
125 and more	80	130 and more (38) + 23 = (61)
130 and more	61	135 and more 20 + 18 = 38
135 and more	38	140 and more (7) + 13 = (2)
140 and more	20	
145 and more	7	145 and more ① + 7 = ⑦
150 and more	zero	150 and more (i)

To represent this table graphically, follow the steps of representing the ascending cumulative frequency to get the following graphical representation:

Unit 3: Lesson 2



Exercises (3-2)

🦺 Below are the scores of 100 students in an experimental Maths exam.

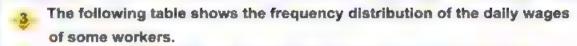
Sets	0-	10-	20-	30-	40-	50-	Total
Frequency	8	14	15	28	23	12	100

Required:

- Form both the ascending and descending cumulative frequency tables
- Graph both the ascending and the descending cumulative frequency curves on the same graph paper.
- From the graph, find the number of students who got less then 40 marks and those who got 40 marks and more.
- Find the percentage of success, given that the minimum mark of success is 20 marks.
- What is the percentage of the students who got more than 40 marks?
- The following table shows the frequency distribution of the scores of 50 students in an experimental math exam.

Intervalle	2-	6-	10-	14-	18-	22-	26-	Total
Effectif	3	5	9	10	12	7	4	50

Required: Graph the ascending cumulative frequency curve.



Required. Graph the descending cumulative frequency curves.

The following table shows the frequency distribution of the ages of 50 workers.

Sets	20-	25-	30	35	40-	45-	50-	Total
Frequency	5	8	9	13	****	5	3	50

Required:

The same of the sa

- A Complete the missing space.
- Graph the ascending and descending cumulative frequency curves
- Find: First: the number of workers whose ages are more than 32.

Second: The number of workers whose ages are less than 43.

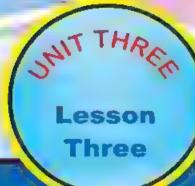
5. The following table shows the frequency distribution of the scores of 1000 students in a final year exam.

Percentages	20-	30-	40	50	60	70-	80	90	Total
Number of students	30	70	160	260	150	130	110	90	1000

Required:

- Graph the ascending and descending cumulative frequency curves.
- Find the number of students whose scores are less than 75 marks
- Find the number of students whose scores are more than 85 marks

Arithmetic Mean, Median and Mode



Think and Discuss

First: the mean

You have learned to find the mean for a set of values and learned that

The arithmetic mean

Example: If the ages of 5 students are 13, 15, 16, 14, and 17 years old, then

The mean of their ages =
$$\frac{13 + 15 + 16 + 14 + 17}{5}$$

= $\frac{75}{5}$ = 15 years

Remark: 15 × 5 = 13 + 15 +16 +14 + 17

The mean; is the simplest and most commonly used type of averages, it's that value given to each item in a set, then the total of these new values is the same total of the original values it can be calculated by adding up all values, then divide the sum by the number of values.

Finding the mean of data from the frequency table with sets. How can you find the mean of the following frequency distribution:

Sets	10-	20 ~	30 -	40 -	50 -	Total
Frequency	10	20	25	30	15	100

Remarks

To find the mean for a frequency distribution with sets, follow the following steps:

You will learn how

- To find the mean from a frequency table with sets
- To calculate the median from a frequency table
- To calculate the mode from a frequency table with sets

key term/

- Mean.
- Median.
- Frequency histogram.
- **∜** Mode

1 Determine the centers of sets:

The center of the first set = $\frac{20+10}{2}$ = 15. The center of the second set = $\frac{30+20}{2}$ = 25 ... and so on

Since the ranges of the subsets are equal and each = 10

We consider the upper limit of the last set = 60 and then :

its center =
$$\frac{50 + 60}{2}$$
 = 55

Porm the following vertical table:

Sets	Centre of the sets (X)	Frequency	Centre of the sets X	×	frequency F
10 -	15	10		150	
20	25	20		500	
30 -	35	25		875	
40 -	45	30		1350	
50 -	55	15		825	
Total		100		3700	

- If the mean of the scores of a student during the first 5 months is 23.8. What is the score of the 6th month if the mean of his scores is 24 marks?
- The following table shows the frequency distribution of the weights of 30 children in kg.

Weight in (kg)	6-	10⊣	14-	18	22-	26-	30-	Total
frequency	2	3	April	8	6	4	2	30

Complete the table, then find the mean of such a distribution

Second: the median

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

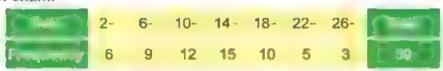
Finding the median of a frequency distribution with sets graphically:

- 1 Draw the ascending or descending cumulative frequency table, then draw the cumulative frequency curve of it.
- 2 Determine the order of the median = The total of frequency
- 3 Determine point A on the vertical axis (frequency) which represents the order of the median.
- Oraw a horizontal straight line from point A to intersect the curve at a point. Form this point, draw a vertical straight line on the horizontal axis to intersect it at a point that represents the median.



Example (1)

The following table shows the frequency distribution for the scores of 60 students in an exam.

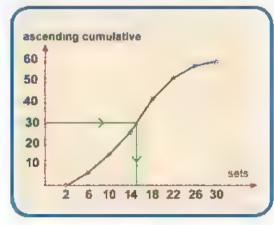


Find the median of the distribution using the ascending cumulative frequency table.

Solution

- 1 Draw an ascending cumulative frequency table
- 2 Find the order of the median = $\frac{60}{2}$ = 30
- Oraw the ascending cumulative frequency curve, and get the median form the graph

The upper limits of the sets	The ascending cumulative frequency
Less than 2	0
Less than 6	6
Less than 10	15
Less than 14	27
Less than 18	42
Less than 22	52
Less than 26	57
Less than 30	60



From the graph, the median = 14.8 mark



Think up Can you find the median using the descending cumulative frequency table? Is the value of the median different in such a case?.



The following table shows the daily wages of 100 workers in a factory..

daily wages in LE (sets)	15-	20-	25-	30-	35-	40-	Total
Number of workers (frequency)	10	15	22	25	20	8	100

Required:

- Graph the ascending and descending cumulative frequency curves on one figure.
- Can you find the median wage from this curve?

Solution

pper boundaries of sets	Cumulative frequency	Lower boundaries of sets	Cumula freque
Less than 15	zero	15 and more	100
s than 20	10	15 and more	90
ss than 25	25	15 and more	75
ess than 30	47	15 and more	53
ess than 35	72	15 and more	28
ess than 40	92	15 and more	8
ess than 45	100	15 and more	zero

Remarks

The ascending cumulative frequency curve intersects with the descending cumulative frequency curve at one point which is m.

Unit 3: Lesson 3

The y-coordinate for the point M = 50

$$=\frac{100}{2}$$

- = the order of the median
- .. The X-coordinate of the point M determines the median

every 10 mm of the x coordinate represents L.E 5

Complete: 2 mm represents

The median wage = $30 + \frac{2 \times 5}{10} = LE 31$.





Draw the descending cumulative

Frequency
1000
80
70
60
50
40
30
20
10
15 20 25 30 35 40 45 Sets

frequency curve for the following frequency distribution, then find the value of the median.

Sets	5	10 -	15	20	25	30 -	total
Frequency	4	6	10	17	10	3	50

Third: the mode

The mode is the most common value in the set or in other words, it is the value which is repeated more than any other values.



The following table shows the frequency distribution for the scores of 40 students in an examination.

Sets	2-	6-	10-	14_	18-	22-	26-
Frequency	3	5	8	10	7	5	2

Find the mode of this distribution graphically

Solution

You can find the mode of this distribution graphically using the histogram as follows: First: draw a histogram.

Draw two perpendicular axes one horizontal to represent sets and the other vertical to represent the frequency of each set.

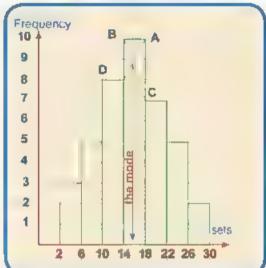
- 2 Divide the horizontal axis into a number of equal parts using a suitable drawing scale to represent sets
- Orvide the vertical axis into a number of equal parts using a suitable drawing scale such that the greatest frequency among sets can be represented.
- Oraw a rectangle whose base is set (2-) and height is equal to the frequency (3).
- 6 Draw another rectangle adjacent to the first one whose base is set (6-) and height is equal to the frequency (5).
- 6 Repeat drawing the rest of adjacent rect-

angles till the last set (26-).

Second: Finding the mode from the histogram, to find the mode from the histogram, we observe that: the most repeated set is (14-), and it is called the mode set, why?

Define the intersection point of AD , BC from the graph, and from this point, drop a vertical line on the horizontal axis to define the sequential value within that distribution

From the graph, what's the mode value?



Exercises (3 - 3)

1 The following table shows the frequency distribution of 50 workers days-off:

Sets	2	6-	10-	14-	18-	22-	26-
Frequency	4	5	8	K-2	7	5	1



Contract of the second

the value of K

The Arithmetic mean for that distribution.

The following table shows the frequency distribution of the heights of 120 students in centimeters:

Height in (cm)	140	144-	148-	152-	156-	160-	total
Frequency	12	20	38	22	17	11	120

Find the mean.

3 The following table shows the frequency distribution of 50 workers' wages in a factory:

sets of wages	300-	400-	500-	600-	700-	total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve, then find the median.

From the following frequency table with equal sets in range.

Sets	10-	20-	30-	40-	×-	60-	total
Frequency	12	15	25	27	k ± 4	4	100

- The value of both X and K.
- Graph the ascending and descending cumulative Frequency curves in one figure, then find the median.
- 5 The following table shows the frequency distribution of the weights of 50 students in Kg.



- the value of K.
- Graph the frequency histogram, then find the mode weight
- The following table shows the frequency distribution of the height of 200 students.

Height in (cm)	110-	115	120-	125-	130-	135-	140-	total
Number of students	10	12	28	35	60	40	40	200

Graph the frequency histogram, then find the mode length.

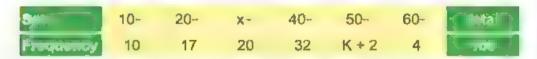
General Exercises

The following table shows the frequency distribution for the scores of 50 students in an examination:

Sets	2-	6 -	10-	14-	18-	22-	26-	total
Frequency	3	5	9	10	12	7	4	50

Find First: the mean of the student's score. Second: The median

From the following frequency table with equal sets in range, find:



First: find the value of X and K

Second: graph the ascending and descending cumulative curves on one figure, then calculate the median

Find the mode of the following frequency distribution for the scores of 40 student in an examination:

The following table shows the frequency distribution with equal - range sets for the weekly wages of 100 works in a factory.

Sets of wages in L.E 70 80- 90- 100- x 120- 130-Number of workers 10 13 f-4 20 16 14 11

Trouver

The value of x and F

B. The mode of wages in L.E.

Activity

The following table shows the frequency distribution for the weights of 50 students in K.g at a school.

Weight in K.g	30 ~	35 -	40 -	45-	50-	55-	total
Number, of students	7	3 k	4k	10	8	4	50

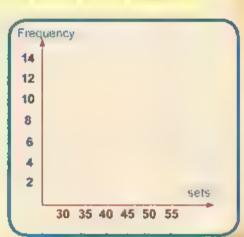
First: find the value of K.

Second: calculate the mean.

Third: Draw the ascending cumulative frequency curve.

Fourth: Draw the histogram and find the mode of weights.

Fifth: Find the median.



Unit test

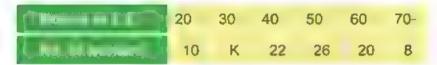
Complete the following:

- A If the lower limit of a set is 8 and the upper limit of the same set is 14, then its centre is
- B If the lower limit of a set is 4 and its center is 9, then its upper limit is
- The intersection point of the ascending and descending cumulative frequency curves determines on the sets axis
- The location of the top of the frequency curve on the set axis is
- If the mean of a frequency distribution is 39.4 and the total of its frequency is 100, then the total of the product of multiplying each set frequency by its centre =
- 2 The following table shows the frequency distribution of Weights of 20 children in k.g

sets	5	15 -	25 =	35	45	total
Frequency	3	4	7	4	2	20

Find the median weight in k g using the ascending and descending cumulative frequency curve of this distribution.

3 below is the frequency distribution of the weekly bonus of 100 workers in a factory.

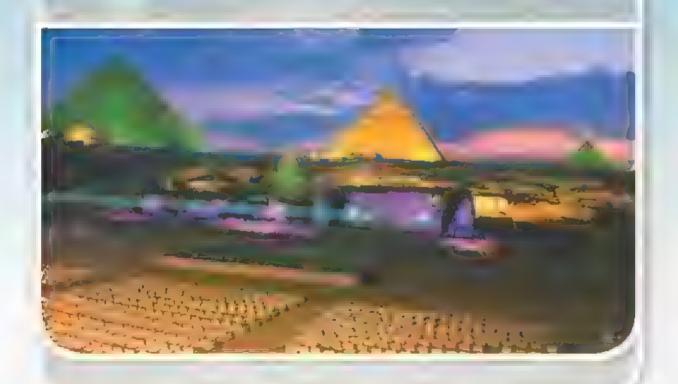


- A Calculate the value of K.
- B. Find the mean of this distribution.
- G The mode value of the weekly bonus using the histogram.

UNIT FOUR

4

Geometry





Medians Of Triangle

Think and Discuss

You will learn how

- Medians of the triangle.
- ¹; A 30°− 60° − 90° triangle

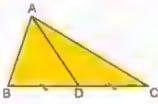
key terme

- Median of the thangle
- Point of Concurrence

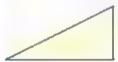
The medians of a triangle is the line segment drawn from the triangle vertex to the middle of the opposite side of this vertex.

ABC is a triangle where the point D bisects BC.

So AD is a triangle Median.



- O How many medians does the trianglle have?
- Draw the medians in each triangle.





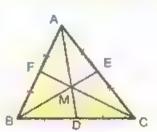




Theorem 1

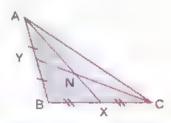
The medians of a triangle are concurrent

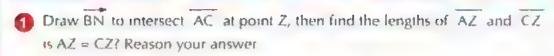
ABC is a triangle where point D bisects BC, point E bisects AC, point F bisects AB, then AD, BE and CF all intersect in one point (M)





In the Figure opposite: ABC is a triangle where point x bisects \overline{BC} , point y bisects \overline{AB} , and $\overline{AX} \cap \overline{CY} = \{N\}$.





Measure, then complete.



Theorem 2

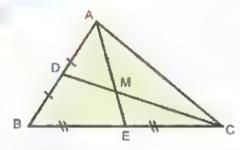
The point of concurrence of the medians of the traingle divides each median in the ratio of 1/2 from its base



Practice



Complete



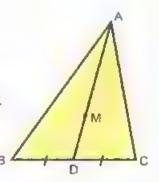
$$ME = 3cm$$
, $MC = 8cm$

$$ME = AE, MC = .$$
 CD

Fact

AD is a median in
$$\triangle$$
 ABC, $M \in \overline{AD}$ if $AM = 2 MD$,

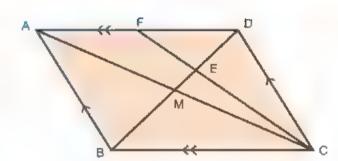
then M is the point where the medians of the triangle intersect.













Example (1)

In the figure opposite: ABCD is a parallelogram where its two diagonals intersect at point M, point E E DM and DE = 2 EM.

C E is drawn and intersected AD at point F.

Prove that: AF = FD

Proof: In the parallelogram ABCD



Theorem 3

In the right - angled triangle, the length of the median from the vertex of the oght angle equals half the length of the hypotenuse

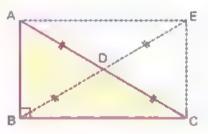
Given data: ABC is a triangle where m ($\angle B_1 = 90^\circ$, BD is a median in A ABC.

Required: Prove that: $BD = \frac{1}{2}AC$.

Construction : Draw BD, let point E & BD where BD = DE.



in the Figure ABCE, AC , BE bisect each other.





... the Figure ABCE is a parallelogram.

A ABCE is a rectangle.

$$BE = AC.$$

$$v BD = \frac{1}{2} BE$$

$$\frac{1}{2}$$
 BD = $\frac{1}{2}$ BE ∴ BD = $\frac{1}{2}$ AC



The converse of the theorem 3

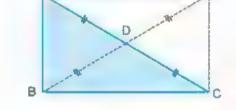
If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right

Given: ABC is a triangle where BD is a median,

$$BD = DA = DC$$

Required: Prove that: m (∠ ABC) = 90°.

Construction : Draw BD and let point E & BD where BD = DE.



Proof: \therefore BD = $\frac{1}{2}$ BE = $\frac{1}{2}$ AC.

In the Figure ABCE, AC and BE are equal in length and bisect each other.

.. the Figure ABCE is a rectangle.

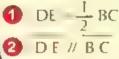


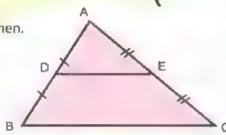
Corollary

The ineight of the nick proposite in the angle of 30° in the right - angled triangle equals half the length of

Remember:

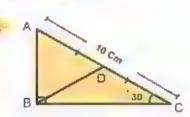
If the point D bisects AB and the point E bisects AC, Then.





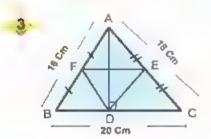
Exercises (4-1)

Complete:

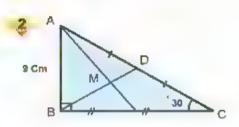


$$BD = \dots cm, AB = \dots cm$$

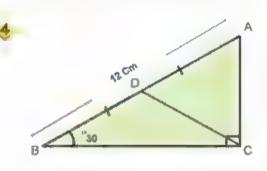
Perimeter $\triangle ABD = \dots cm$



$$DF = ... cm, DE = ... cm,$$
 $FE = ... cm$
 $Perimeter \triangle DEF = ... cm$



AC	cm, BD ~	Can
MD=	BD, MD -	CHY



$$AC = \dots$$
 cm, $AD = \dots$ cm
 $BC = \dots$ cm, $CD = \dots$ cm

· 🐠 🗆

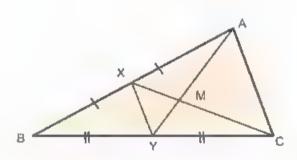
In the figure opposite:

ABC is a triangle, X bisect AB,
Y bisect BC, XY = 5 cm,
X C n AY = {M}
where: CM = 8cm, YM = 3 cm



Find:

- (1) The perimeter of \triangle MXY
- (2) The perimeter of △ MAC

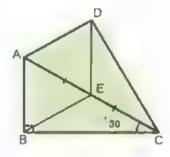




- S ABC is a triangle where point D bisects BC, and point M∈ AD. AM = 2 MD. Draw CM to intersect AB at point E.
 If EC = 12 cm,
 then find the length of E M
- In figure opposite:

ABC is a right-angled triangle at B, $m (\angle ACB) = 30^{\circ}$.

AB = 5 cm, point E is the mid point of \overline{AC} . If DE = 5 cm, then prove that m ($\angle ADC$) = 90°.





The Isosceles Triangle

Think and Discuss

You will learn how

- To define the properties of the isosceles triangle.
- To define the classifications of the isosceles triangle...

key termi

- The isosceles
- The equilateral triangle.
- % The scalene triangle

You have learnt that triangles are classified according to the lengths of their sides into three types:

The scalene triangle (two sides are congruent) AB # BC AB # AC C The isosceles triangle (three sides are congruent) The equilateral triangle (three sides are congruent)

In the figure opposite:

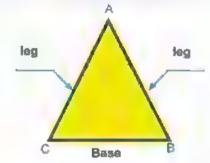
BC# AC

Remaides the two sides AB, AC are congruent (of equal

AB=AC

lengths), so the triangle ABC is called isosceles triangle while the point A is called the vertex.

B C is the base, and the two angles B and C are the base angles of the triangle.



AB=AC=BC

The properties of isosceles triangle

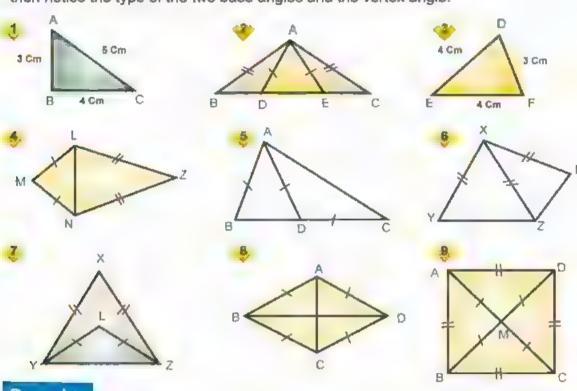
In any isosceles triangle

- O What is the type of the base angles? (acute right obtuse)
- O What is the type of the vertex angle?



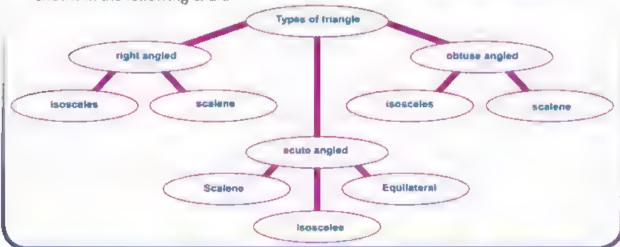
Practice

In each of the following figures, state the isosceles triangles and define their bases, then notice the type of the two base angles and the vertex angle.



Remark

- 1 Both of the base angles in the isosceles triangle are acute
- 2 The vertex angle in the isosceles triangle can be either acute, right or obtuse. So, the isosceles triangle can be either obtuse, right or acute angled triangle as shown in the following chart:





The Isosceles Triangle Theorems

Think and Discuss

You will learn how

- To define the relation between the base angles in the isosceles triangle.
- To define the relation among the measures of the angles in the equilateral triangle.
- To define the relation between two sides opposite to two equal angles in a triangle.
- To know that if the angles in a triangle are congruent, then the triangle is equilateral

key terms

- The isosceles triangle
- % The base angles

Is there a relation among the measures of the two base angles in the isosceles triangle?

to know that, let's conduct the next activity:



Activity

Using the compass

Draw several isosceles triangles
 as shown in the opposite figure
 Where AB = AC



- Find using a protractor, the measure of the two base angles ∠ ABC and ∠ ACB
- Write down the data you got in a table as follows, then compare the measures in each case.



Keep your activity in the portfolio



Theorem 1

(the isosceles triangle theorem) the base angles of the isosceles triangle are congruent.

Given: ABC is triangle in which AB = AC

R.T.P: 2 B = 2 C



Proof: The two triangles ADB and ADC are right angled in which.

from the congruency , we deduce that

(a given)

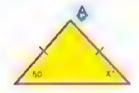
(a common side)

(a hypotenuse and a side)



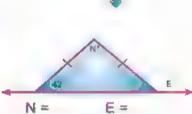


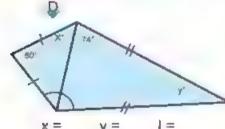
1 In each of the following figures, find the value of the symbol that is used to measure the angle:



x =



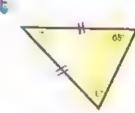




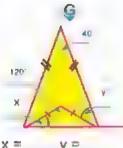
y = . 1 =

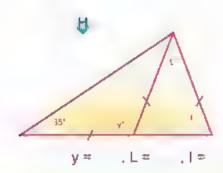


M =L =



. | = L =





First Term - El-Fath Press



In the figure opposite, ABC is an isosceles triangle in which AB = AC

 $m(\angle A) = 40^{\circ}$, $D \in CB$, $E \in BC$.

First: Find m (2 ABC)



Second: Prove that ABD = ACE



Think: Are the supplementary angles to congruent angles congruent?



Corollary

If the triangle is equilateral, then it is equiangular where each angle measure 60



Example (1)

In the figure opposite: ABC is an equilateral triangle.

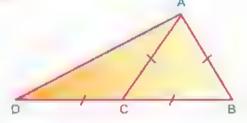
 $D \in BC$ such that BC = CD



prove that AB = AD

Given: AB = BC = CA = CD, $D \in BC$

R.T.P: Prove that 8A = AD



$$\therefore$$
 m (\angle ACB) = m (\angle BAC) = m (\angle B) = 60° (corollary)

∴ ∠ BCA is an exterior angle of the △ ACD

$$m (\angle BCA) = m (\angle CAD) + m (\angle CDA) = 60^{\circ}$$
 (1)

In △ ACD

$$: CA = CD \qquad :: m (\angle CAD) = m (\angle CDA) \qquad (2)$$

from (1), (2) we deduce that: $m (\angle CAD) = m (\angle CDA) = 30^{\circ}$



$$v \cdot m (\angle BAD) = m (\angle BAC) + m (\angle CAD)$$

$$\wedge$$
 m (\angle BAD) = 60° + 30° \approx 90°

Remarks

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non - adjacent interior angles.



Example (2)

In the figure opposite: AB = AD, BC = CD



Given: AB = AD, BC = CD

R.T.P: prove that ∠ABC = ∠ ADC

Proof: in A ABD

$$...AB = AD$$

$$\therefore m \ (\angle ABD) = m \ (\angle ADB) \tag{1}$$

in A CBD

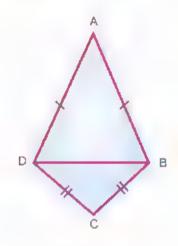
$$\cdot CB = CD$$

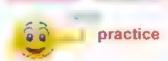
By adding (1) and (2) we deduce that

$$m (\angle ABD) + m (\angle CBD) = m (\angle ADB) + m (\angle CDB)$$

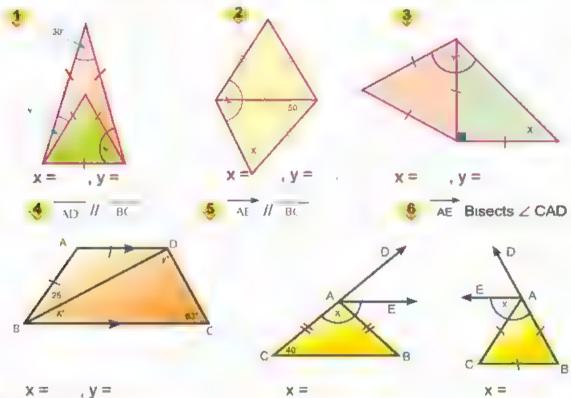
$$\therefore$$
 m (\angle ABC) = m (\angle ADC)

Q.E.D.





In each of the following figures, find the value of the symbol that is used to measure the angle:



Activity

Draw the triangle ABC in which BC = 7 cm, m (\angle B) = m (\angle C) = 50°, then measure the lengths of both \overline{A} B and \overline{A} C. Repeat the activity using other measures for the length of \overline{B} C and the measures of angles B and C, then fill in the table:

Number of the triangle	ВС	m (z B)	m (_ C)	AB	AC
1	7cm	50°	50°		
2					
3					
4					

Are AB and AC equal in length?

- IS AB = AC?
- How can you explain such corollaries geometricaly?



D



Theorem (2)

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles

$$\triangle$$
 ABC, \angle B \equiv \angle C

R.T.P: Prove that: AB = AC

Construction: bisect ∠ BAC with the bisector AD to intersect BC at D



$$\wedge$$
 m (\angle B) = m (\angle C)

$$z \cdot m (\angle BAD) = m (\angle CAD)$$

the sum of the measures of interior angles of a triangle is = 180 °

$$m (\angle ADB) = m (\angle ADC)$$

In the two triangles ADB, ADC

$$m (\angle BAD) = m (\angle CAD)$$

$$m (\angle ADB) = m (\angle ADC)$$

$$\Delta ADB = \Delta ADC$$

Form the congruency, we deduce that AB = AC

Therefore, A ABC is an isosceles triangle



Corollary

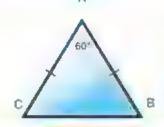
The major of the language of t

In the figure opposite ABC is an isosceles triangle in which

$$AB = AC$$
, $m (\angle BAC) = 60^{\circ}$



ΔABC is triangle

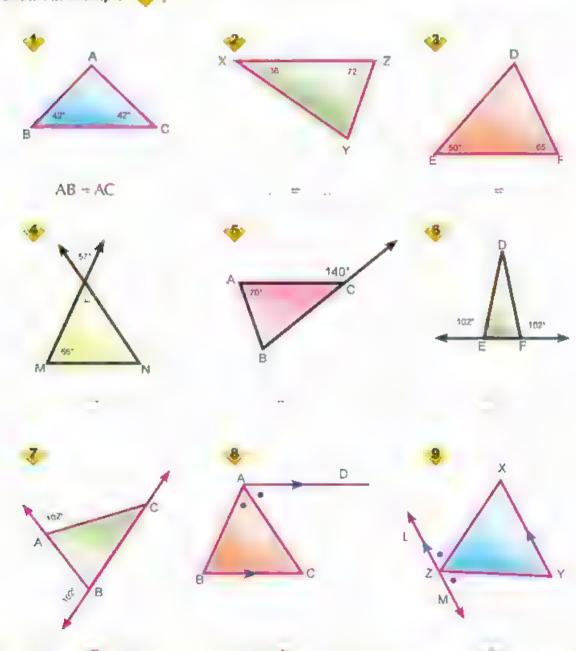




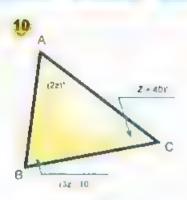
In an isosceles triangle, if any angle has a measure of 60°, then the triangle is an equilateral triangle.

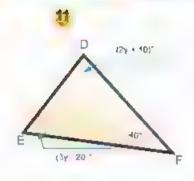


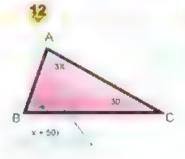
In each of the following figures, define the triangle's sides that are equal in length as shown in example 4:













Examples

In the figure opposite: ABC is a triangle in which AB = AC, XY // BC



prove that AXY is an isosceles triangle

Given: AB = CA, XY // BC .

Required: prove that AX = AY

Proof: In A ABC AB = AC



The same XY // BC , AC a transversal

∴ m (∠AYX) = m (∠ACB) correspondingly (3)

from (1), (2), (3) we deduce that:

$$m (\angle AXY) = m (\angle AYX)$$

In A AXY

$$\cdot \cdot m (\angle AXY) = m (\angle AYX)$$

: AX = AY

i.e. the triangle AXY is an isosceles triangle

Q.E.D

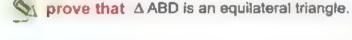


Think: Can we deduce that XB = YC ? Explain your answer,



ABC is a right angled triangle at B, m (∠ C) = 30°,

D ∈ AC where DB = DC



Given: m (∠ A B C) = 90°, m (∠ C) = 30°, D B = D C

R.T.P: prove that A B = B D = AD

proof: In A DBC + DB = DC

$$\therefore$$
 m (\angle D B C) = m (\angle C) = 30°

in \triangle ABC \sim m (\angle ABC) =90°, m (\angle DBC) = 30°

$$\therefore$$
 m (\angle BAD) =90 - 30 = 60°

∴ ∠ ADB is an exterior angle of △ B D C

$$\therefore$$
 m (\angle A D B) = m (\angle DBC) + m (\angle DCB)

$$m (\angle ADB) = 30^{\circ} + 30^{\circ} = 60^{\circ}$$

In \triangle ABD \sim the sum of the measures of the interior angles of a triangle = 180°

(4)

(2)

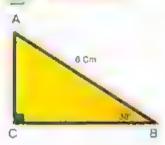
$$\therefore$$
 m (\angle ABD) = 180° - (60° + 60°) = 60° (3)

from (1), (2), (3)
$$\therefore$$
 m (\angle ABD) = m (\angle ADB) = m (\angle A)

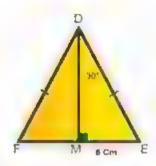
i.e.
$$\angle ABD = \angle ADB = \angle A$$

Exercises (4-3)

Complete:







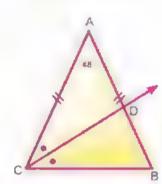
DE = cm, m (
$$\angle$$
E) = .°
EF = cm, m (\angle MDF) = °

In the figure opposite :

AB = AC, $m (\angle BAC) = 48^{\circ}$ CD bisects ∠ B C A and intersects AB at D



Find m (Z B) et m (Z BCD)

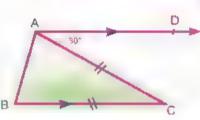


In the figure opposite:

A B C is a triangle in which AC = BC. AD // BC , m (DAC) = 30°



Find The measures of the angles in A ABC

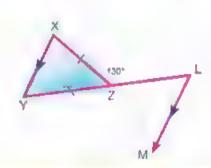


🕪 In the figure opposite :

ZE LY , X Z = Y Z $m (\angle L Z X) = 130^{\circ}, LM // XY$.



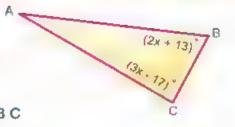
Find m (Z MLY)





In the figure opposite :

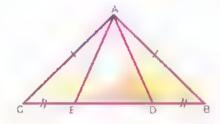
$$AB = AC$$
, $m(\angle B) = (2x + 13)^{\circ}$
 $m(\angle C) = (3x - 17)^{\circ}$



- Find the measures of the angles of Δ A B C
- 6 In the figure opposite:

A B C is an isosceles triangle in which A B = A C,

D ∈ BC , E ∈ BC such that BD = E C



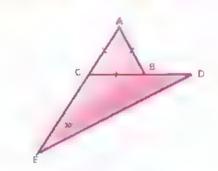
Prove that First: A ADE is an isosceles triangle.

Second: ∠ADE=∠AED

In the figure opposite: ABC is an equilateral traingle.

E ∈ AC, D ∈ CB,

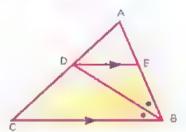
m (∠ DEC) = 30°



- Prove that A DCE is an isosceles triangle.
- In the figure opposite:

 BD bisects ∠ A B C and intersects A C at D,

 DE // BC where E ∈ AB.



- Prove that Δ EBD is an isosceles triangle.
- ABC is a triangle in which D∈ AB,

 E∈ BC such that BD = BE, So if DE // AC



- ABC is a triangle in which AB = AC,

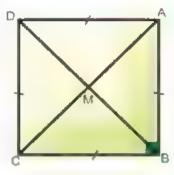
 BD bisects ∠ABC, CD bisects ∠ACB, BD ∩ CD = {D}
- Prove that \triangle DB C is an isosceles triangle.



Complete and discuss.

A In AABC, m (∠ABC) =

$$\pm m (\angle BAC) = m (\angle BCA) = -1$$



- Does the diagonal A c bisect ∠A?
- D Does the diagonal BD bisect both ∠B and ∠D?
- Is A M A D an Isosceles triangle? why?
- State those isosceles triangles whose vertices are point M.
- Is M the midpoint of Ac and BD?

From the previous clauses, deduce the properties of the square and keep them In



Corollaries of isosceles triangle theorems

Trink and Discuss

You will learn how

the theorems of sosceles triangles

key term/

- The isosceles triangle
- The bisector of a vertex angle
- The bisector of a triangle base
- The axis of symmetry for a line segment.

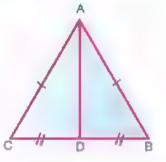
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Corollary (1)

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the figure opposite:

in \triangle ABC , AB = AC, AD is a median then: AD bisects \angle BAC. AD \bot BC



Remarks

 $\triangle ADB \equiv \triangle ADC$. Why?



Corollary (2)

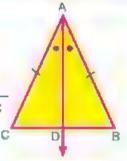
The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the figure opposite:

In AABC, AB=AC,

AD bisects ZBAC

then D is a midpoint of BC and AD L BC



Romarkiy

 $\triangle ADB = \triangle ADC$. why?



Corollary (3)

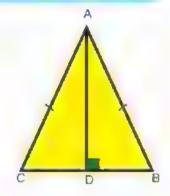
The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the figure oppasite:

In
$$\triangle$$
 ABC, AB = AC, AD \bot BC
then D bisects BC, m (\angle BAD) = m (\angle CAD)



ΔADB≡ΔADC why?

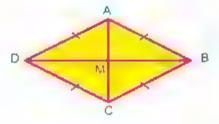




Think

In the figure opposite:

ABCD is a quadrilateral in which all sides are equal in length, this figure is called rhombus, its diagonals are AC and BD, they intersect at point M



Remarks

 $\triangle ABD = \triangle CBD$ why?

$$\therefore m (\angle ABD) = m (\angle CBD)$$
in $\triangle ABC$, $AB=BC$, BM bisects $\angle ABC$

Are the two diagonals of the rhombus perpendicular?

Do the two diagonals of the rhombus bisect each other?

Does the diagonal of the rhombus bisect the vertex angles which it connects?

Write down your answer.

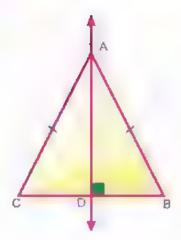
Axes of symmetry

First: axes of symmetry in the isosceles triangle:

The axis of symmetry of the isosceles triangle is the straight line drawn from the vertex angle perpendicular to its base.

In the figure opposite:

 \triangle ABC in which AB = AC, AD \perp BC then AD is the axis of symmetry in the isosceles triangle ABC.



Discuss:

Does the isosceles triangle has more than one axis of symmetry?

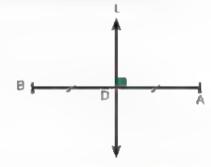
- How many axes of symmetry are there in the equilateral triangle?
- Are there any axes of symmetry in the scalene triangle?

Second: Axis of symmetry of a line segment:

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment in brief it is known as the axis of a line segment.

In the figure opposite:

If D the midpoint of AB and
The straight line L ⊥ AB
Where D ∈ L, then the straight line L
Is the axis of AB



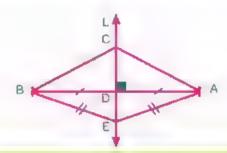


Important property

Any point at the axis of symmetry of a line segment is at equal distances from its end points.

Romarie

- 1 If C ∈ L then AC = B C
- ② If EA = E B then E ∈ L. why?







Examples

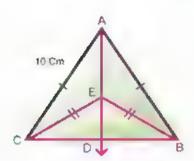


In the figure opposite

$$AB = AC = 10 \text{ cm}$$
, $EB = EC$

AE
$$n$$
 8C = $\{D\}$

If BC = 6 cm, find the length of CD and AD



Given: AB = AC, EB = EC

R.T.P : Find CD and A D

Proof: $\forall AB = AC$ $\Rightarrow A$ is on the axis of BC

v E B = E C

E is on the axis of BC

.. AE is the axis of BC

D is the midpoint of BC , AD 1 BC

D is the midpoint of BC, BC = 6cm \therefore CD = 3cm

· AD 1 BC

... In A ADC that is right angled triangle at D

$$(AD)^2 = (AC)^2 - (CD)^2$$

$$(AD)^2 = 100 - 9$$

$$\therefore AD = \sqrt{91} \text{ cm}$$



In the figure opposite

ABC is a triangle in which AB = AC,

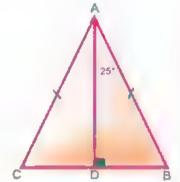
AD
$$\perp$$
 BC , m (\angle BAD) = 25°,

BC = 4 cm Find:





B the length of DC

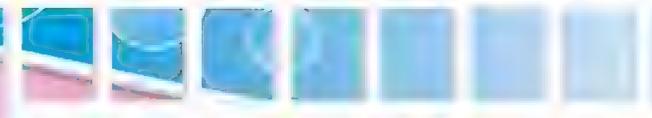


Solution

Given: AB = AC,

AD
$$\perp$$
 BC , m (\angle BAD) = 25°, BC = 4 cm

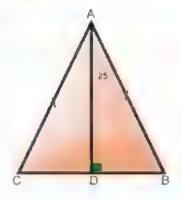
R.T.P: m (Z D A C), and the length of DC.



Proof : in A ABC

$$\therefore$$
 m (\angle DAC) = m (\angle DAB) = 25°,

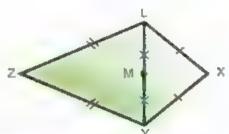
D C =
$$\frac{1}{2}$$
 BC = $\frac{4}{2}$ = 2 cm.



practice



$$xy = xL, Zy = ZL, LM = YM$$



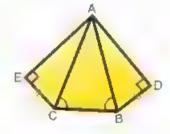
Prove that X, M and Z are on the same straight line

2 In the figure opposite:

$$m (\angle ABC) = m (\angle ACB)$$

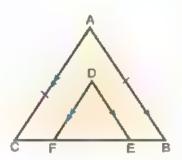
$$m (\angle D) = m (\angle E) = 90^{\circ}$$

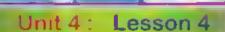
Prove that : $m (\angle DAB) = m (\angle CAE)$



In the figure opposite:

Second: $m (\angle BAC) = m (\angle EDF)$





Exercises (4-4)

Complete to get a correct statement

- A The vertex angle bisector in the isosceles triangle bisects the base and is
- B. The number of symmetrical axes in the equilateral triangle is
- Any point at the axis of a line segment symmetry is at two equal distances from
- If the measurement of an angle in the isosceles triangle is 100°, then the measurement of an angle of the other two = ... °.

Choose the correct answer:

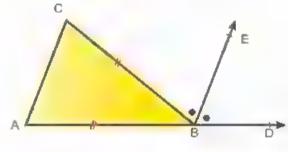
♣ The number of axes of symmetry in the isosceles triangle =

The triangle whose sides lengths are 2 cm, (X + 3) and 5 cm becomes an isosceles triangle when $X = -\infty$ cm.

The intersecting point of the medians of a triangle divides each other from the direction of the base in a ratio

In the figure opposite:

$$AB = BC$$
, BE bisects $\angle CBD$



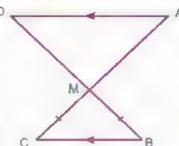
In the figure opposite:

$$AC \cap BD = \{M\}$$

 $AD //BC , MB = MC$

Prove that:

- (1) A AMD is an isosceles triangle
- (2) The axis of symmetry of \triangle AMD is the same of \triangle BMC.



General Exercises

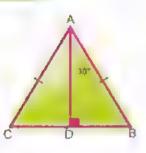


In the figure opposite

First: Find the length of aD , AD .

Second: How many axes of symmetry are there at A ABC?

Third: What is the area of A ABC?



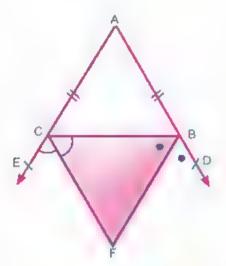
In the figure opposite



Prove that

First: A BFC is an isosceles triangle

Second: AF is the axis of symmetry of BC



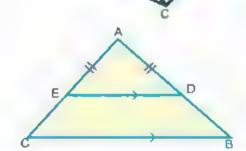
In the figure opposite



Prove that



In the figure opposite

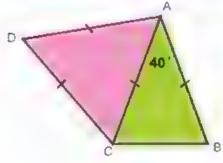


In the figure opposite

$$AB = AC = AD = CD$$

$$m (\angle BAC) = 40$$

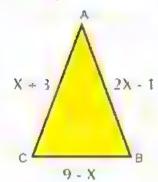
Find: m (∠ BCD)



for the figure opposite

ABC is a triangle which $m (\angle 8) = m (\angle C)$

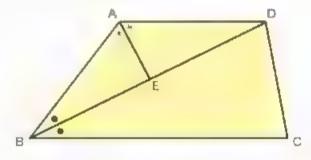
Find: The perimeter of in the triangle



To the figure opposite

ABCD is a quadrilateral in which AD // BC , B D bisects

Prove that

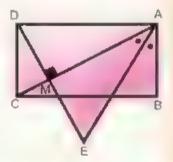




- 1 Using a ruler and a compass, draw the acute-angle ABC. At the opposite side from BA. Draw AE \\ BC
- In the opposite figure ABCD is a rectangle in which AC is a diagonal, AE bisects ∠ B A C,



Prove that DA = DE.



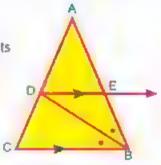


Unit test

- Complete the following to get a correct statement :
 - A The base angles in an isosceles triangle are
 - The median that is drawn from the vertex of an isosceles triangle is and
 - \bigcirc In the triangle ABC, AB = AC, m (\angle A) = 70°, so m (\angle C) =
 - The number of the axes of symmetry in an equilateral triangle #
 - The measure of the exterior angle in an equilateral triangle =
 - The straight line perpendicular to the midpoint of a line segment is called
- in the figure opposite :

ABC is a triangle in which BD bisects ∠ ABC and intersects

Prove that B E = E D

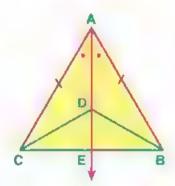


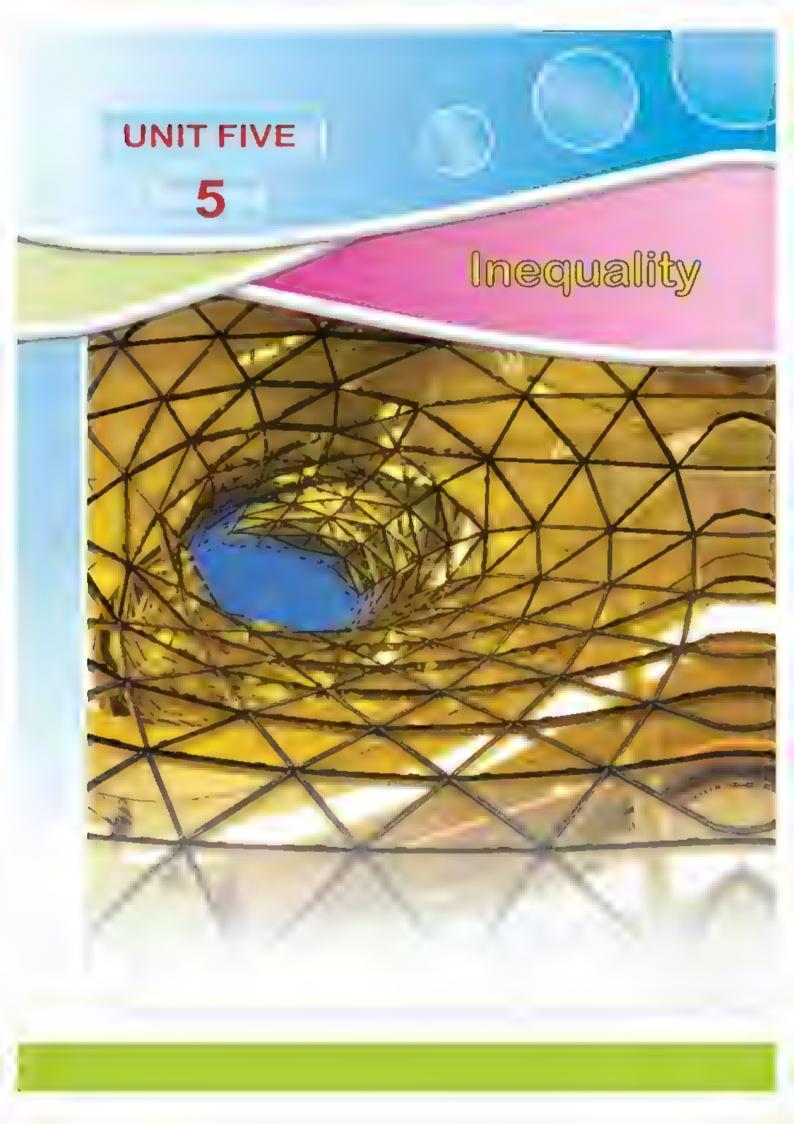
In the figure opposite ABC is a triangle in which

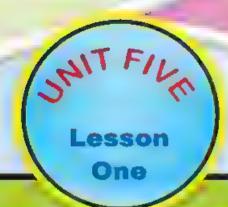
$$AB = AC$$
, AE biscts $\angle BAC$, $AE \cap BC = \{E\}$

Prove that

$$BE = \frac{1}{2} BC$$







Inequality

Think and Discuss

You will learn how

- To define the concept of inequality
- To define axioms of inequality.

key termr

- Inequality.
- & axioms.
- greater than >
- % Less than <
- % equal to

The concept of inequality:

- Do all the students in your class have the same height?
- 2 Are there any differences among the measures of acute, right and obtuse angles?

What does this difference mean?

Remarks

An Inequality means that there is a difference in the heights of the students and in the measures of the angles. This difference is represented by the relation of inequality which is used to compare two different numbers.



Examples

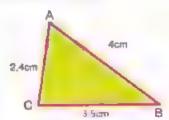
- fi ∠ABC is an acute angle then: m (∠ABC) < 90°</p>
- 2 In the figure opposite, ABC is a triangle in which:

AB = 4cm, BC = 3.5cm,

A C = 2.4cm

then: AB > BC . BC > AC

i.e AB > BC > AC





In the figure opposite, find: $m (\angle ACB)$, $m (\angle ACD)$ and $m (\angle ADE)$ then complete by using > or < :



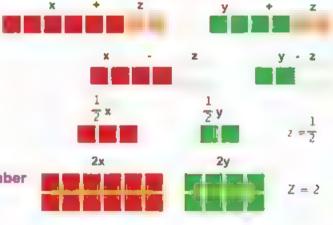
All the previous relations are called inequalities.

Axioms of inequality

For any given three numbers x, y and z:



- - then: x+z>y+z
- ② If: x > y
 - then: x-z>y-z
- If: x>y, z is a positive number then: xz>yz
- If: x > y , y > z
 then: x > z.
- 6 If: x>y, A>B then: x+A>y+B



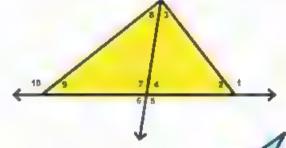
- х у z
- x + A y + B

The measure of any exterior angle of a triangle is greater than the measure of any interior angle except for the adjacent angle.

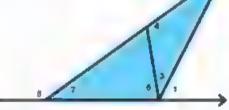


In the figure opposite: which of the following angles has the greatest measure?

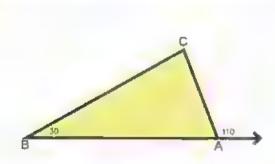


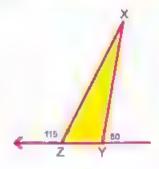


- In the figure opposite , find:
 - All angles of measures less than m (∠ 1)
 - All angles of measures greater than m (∠ 6)
 - C All angles of measures less than m (∠ 4)



Order the measures of the angles in the triangle ABC in an ascending order and the measures of the angles in the triangle X Y Z in a descending order.





$$m(\angle) < m(\angle) < m(\angle)$$

$$m(\angle)>m(\angle)>m(\angle)$$









Example

In the figure opposite:

 $m (\angle ACB) > m (\angle ABC), DB = DC$

Prove that: $m (\angle ACD) > m (\angle ABD)$

Given: m (∠ACB) > m (∠ABC), DB = DC

Required to prove: $m (\angle ACD) > m (\angle ABD)$

R.T.P: .. DB = DC

 $\wedge m (\angle DCB) = m (\angle DBC)$

(1)

 $v m (\angle ACB) > m (\angle ABC)$

(2)

.. By subtracting (1) from (2), we get:

 $m (\angle ACB) \cdot m (\angle DCB) > m (\angle ABC) \cdot m (\angle DBC)$

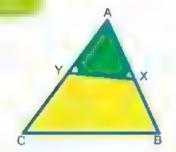
∴ m (∠ACD) > m (∠ABD) Q.E.D

Exercises (5-1)

In the figure opposite :

ABC is a triangle in which AC > AB, $x \in AB$ $y \in AC$, where $m (\angle AXY) = m (\angle AYX)$

Prove that: Y C > X B



In the figure opposite: AB // CD ,

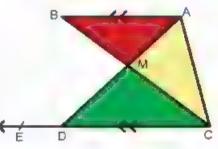
ADn CB = {M}, E e CD , E e CD

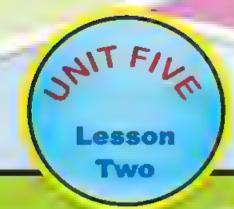
Prove that: m (∠ACD) > m (∠ABC)

m (∠ADE) > m (∠ABC)

M is a point in a triangle ABC ,

Prove that: $m (\angle AMB) > m (\angle ACB)$





Comparing the measures of the angles of a triangle

You will learn how

To compare the measures of angles in a triangle

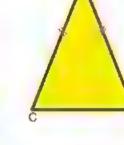
key term/

- % Angle
- Measure of an angle.
- The greatest angle in a triangle
- The smallest angle in a triangle
- The largest side of a triangle.
- The smallest side of a triangle.



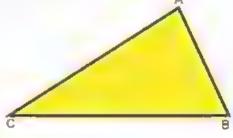
Activity

- 1 In the figure opposite: ABC is an isosceles triangle in which AB = AC
- Fold the triangle to make the vertex B congruent to vertex C. What do you observe regarding to the measures of the angles B, C which are opposite to the two equal sides AC, AB?



- Fold the triangle to make the vertices A, C congruent, what do you observe regarding to the measures of the two angles opposite to the two unequal sides BC, AB?
- Does the difference in the lengths of the two sides in a triangle lead to a difference in the measures of their two opposite angles?
- 2 Draw the scalene triangle. ABC Flip the triangle to make the vertex A coincide the vertex B. What do you observe regarding to the

regarding to the measures of the two angles A, and B that are opposite to the two unequal sides, BC,



Repeat the previous steps to make the vertex B coincide vertex C, what do you observe?



Notice that: In a triangle, if the sides are unequal in length, the measures of the opposite angles are unequal.



Draw the scalene triangle ABC, then measure the lengths of its 3 sides and the measures of the opposite angles, then complete the following table::

Len	gths of sides	Measures of the opposite angles
AB =	cm	m (C) =
BC =	cm	m (Å) =
CA =	cm	m (B)=

What do you observe?



Theorem (3)

(Angle - Comparison Theorem)

In a triangle, if two sides have unequal lengths, then the longer is opposite to the angle of the greater measure

Given: ABC in which AB > AC

R.T.P: m (ACB) > m (ABC)

Construction: take D ∈ AB where AD = A C

proof: in \triangle ACD, AD = AC

v m (ACD) = m (ADC) (1)

A DC is an exterior angle of A BDC

m(ADC) > m(B) (2)

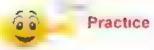
from (1), (2)

m (ACD)>m (B)

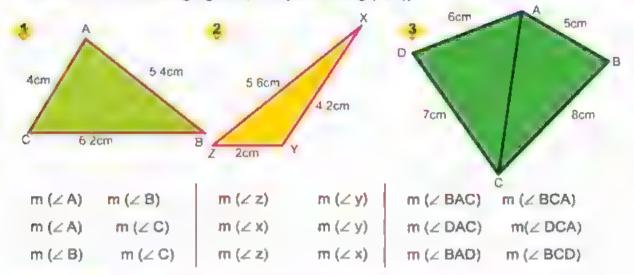
m (ACB) > m (ACD)

m(ACB)>m(ABC)

Q.E.D



In each of the following figures, complete using (>, <))



Remarks

The measure of the greatest angle in the triangle > 60°

The measure of the smallest angle in the triangle is < 60° why?



Example

In the figure opposite:

ABC is a triangle in which AB > BC > CA

Prove that: $m (\angle C) > m (\angle A) > m (\angle B)$

Given: AB > BC > CA

R.T.P: $m (\angle C) > m (\angle A) > m (\angle B)$

Proof: In AABC

 $\triangle AB > BC$ $\triangle m (\angle C) > m (\angle A)$ (1)

 \therefore BC>CA \therefore m (\angle A) > m (\angle B) (2)

from (1), (2) and using the axioms of inequality:

 $m(\angle C) > m(\angle A) > m(\angle B)$

C



Unit 5: Lesson 2

In a triangle, the longest side in length is opposite to the greatest angle in measure while the shortest side in length is opposite to the smallest angle in measure.



Example

In the figure opposite:

ABC is a triangle where BM bisects ∠ABC, and CM

bisects ∠ ACB If: M C > MB

Prove that: m (∠ ABC) > m (∠ ACB)

Given: BM bisects ∠ABC, CM bisects ∠ACB, MC > MB.

R.T.P: Prove that $(\angle ABC) > m (\angle ACB)$

Proof: In A MBC

· MC > MB \therefore m (\angle MBC) > m (\angle MCB) (1)

In A ABC

• BM bisects ∠ ABC ∴ m (∠ MBC) = $\frac{1}{2}$ m (∠ ABC) (2)

 \checkmark CM bisects \angle ACB \therefore m (\angle MCB) = $\frac{1}{2}$ m (\angle ACB) (3)

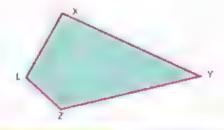
from (1), (2), (3): $\frac{1}{2}$ m (\angle ABC) > $\frac{1}{2}$ m (\angle ACB) Using the axioms of inequality

 \therefore m (\angle ABC) > m (\angle ACB) Q.E.D

Exercise (5-2)

- ∆ ABC in which AB = 2 7cm, BC = 8 5cm, AC = 6cm. Order the measures of the angles of the triangle ascendingly.
- In the figure opposite X Y > X L, Y Z > Z L

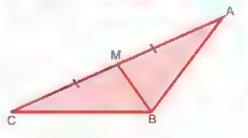
 Prove that: m (∠ XLZ) > m (∠ XYZ)



In the figure opposite :

BM is a median in the triangle ABC, BM < AM

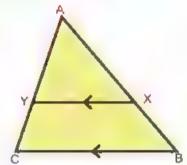
Prove that :∠ ABC is an obtuse angle.



In the figure opposite :

ABC is a triangle , AB > AC XY // BC

Prove that: m (∠ AYX) > (∠ AXY)



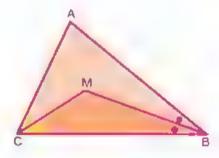
5 In the figure opposite:

ABC is a triangle BM bisects

∠ABC.

C M bisects ∠ ACB.if AB > AC

Prove that : $m (\angle MCB) > m (\angle MBC)$



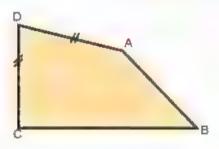
In the figure opposite :

ABCD is a quadrilateral in which AD = DC,

BC > AB

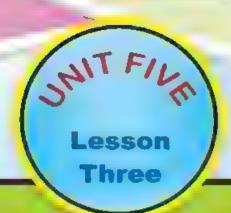
Prove that:

 $m (\angle A) > m (\angle C)$



7 ABCD is a quadrilateral in which AB is the longest side in length. co is the shortest one. Prove that m (∠ BCD) > m (∠ BAD)

Comparing the lengths of sides of a triangle

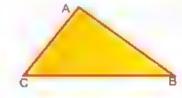


Ininic and Discussi



Activity 1 The figure opposite: ABC is a thangle of unequal measures of angles.

Fold the triangle to make the vertex A coincide vertex B what do you observe regarding to the lengths of the two sides BC and AC, which are opposite to the two unequal angles A and B?



the lengths of the opposite sides

- Repeat the same previous steps to make vertex B congruent to vertex C. What do you observe?
- % When vertex C is coincide to vertex A, what do you observe?
- Are there any equal sides in lengths in that triangle?

Remerk

If the measures of the angles in a triangle are unequal, then the lengths of its sides which are opposite to the angles are unequal.



Activity 2 Draw the triangle ABC where its angles are

unequal in measure then measure the lengths of opposite sides to the angles and complete the following table:

the measures of the angles

			_	
m (∠ A)	= '	BC	= cm	
m (🚄 B)	= .	CA	= cm	
m (∠ C)	# 1	AB	= cm	

What do you observe?

- Is the greatest angle in measure opposite to the longest side in length? Is the smallest angle in measure opposite to the shortest side in length?
- Is it possible to order the lengths of the sides in the triangle in an ascending or descending order in terms of the measures of the opposite angles?

You will learn how

To compare the measures of sides in a triangle

key termi

- The longest side of a triangle
- The shortest side of a triangle
- the greatest angle of a triangle
- the smallest angle of a triangle
- The perpendicular line segment

Theorem (4)

(Side - Comparison Theorem)

in a triangle of two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

Given: $\ln \Delta ABCm(\angle C) > m(\angle B)$

R. T. P: AB>AC

Proof: .. AB , AC are line segments

... one of the following cases should be verified:

(2) A B - A C

(3) A B > A C

If not AB > AC

Either AB = AC AB < AC

if AB = AC, then $m (\angle C) = m (\angle B)$

Again this contradicts the given where $m (\angle C) > m (\angle B)$

and if A B < A C, then m (\angle C) < m (\angle B). According to the theorem above.

Again this contradicts the given, where

$$m (\angle C) > m (\angle B)$$

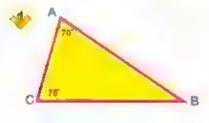
: AB>AC

QED



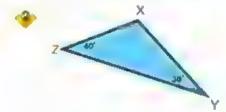
practice

In the following figures, complete using >, < or =:







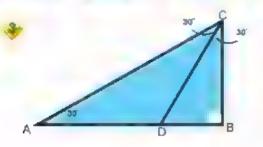


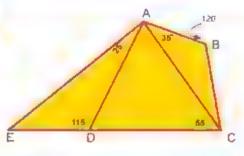
. .. . XZ XY

.... XY YZ

YZ XZ

Unit 5: Lesson 3





Corollaries:



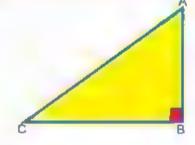
Corollary (1)

- nor i miles i male l'- norman i l'i longest side

In the figure opposite ABC is a right - angled triangle at B.

$$\therefore$$
 m (\angle B) > m (\angle A)

$$\therefore$$
 m (\angle B) > m (\angle C)



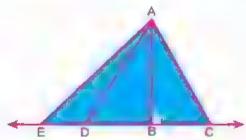
Remarks

In the obtuse angled triangle, the side opposite to the obtuse angle is the longest side in the triangle



Let's think

Is the length of the right leg in the right angled-triangle is shorter than the length of the hypotenuse? Why?





Corollory (2)

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment

Definition:

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from the point to the given line.



Example

in the figure opposite: ABC is a triangle, E ∈ BA

$$m (\angle DAE) = 75^{\circ}$$

Prove that : AC > AB

Given that: AD // BC , m (\angle EAD) = 75°, m (\angle DAC) = 35°

R.T.P: AC>AB

Proof: .. AD // BC . AB is a transversal

∴ m (
$$\angle$$
 B) = m (\angle E A D) = 75*

∴ m (
$$\angle$$
 A C B) = m (\angle D A C) = 35°

alternate angles (2)

Corresponding angles (1)

From (1) and (2):

in AABC

$$m (\angle A B C) = 75^{\circ}, m (\angle A C B) = 35^{\circ}$$

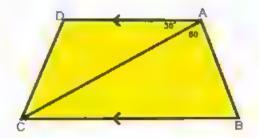
i.e.
$$m (\angle ABC) > m (\angle ACB)$$

Q.E.D



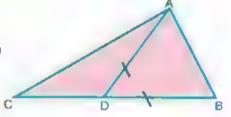
Exercise (5-3)

- \triangle A B C in which m (\angle A) = 40°, m (\angle B) = 75°, Order the lengths of sides of the triangle descendingly.
- In the figure opposite :



In the figure opposite :

in $\triangle ABC$ is a triangle, $D \in BC$ where BD = AD prove that : BC > AC



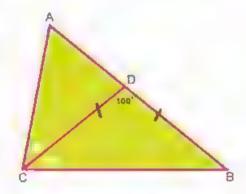
In the figure opposite :

ABC is a triangle, CD bisects∠C and Intersects

AB at point D.

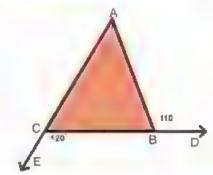
$$m (\angle B DC) = 100^{\circ}, D B = D C$$

Prove that : A C > D B .



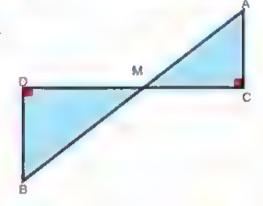
In the figure opposite :

ABC is a triangle, $D \in CB$, $E \in AC$ $m (\angle ABD) = 110^{\circ}, m (\angle BCE) = 120^{\circ}$ Prove that: AB > BC.



In the figure opposite:

prove that : AB > CD

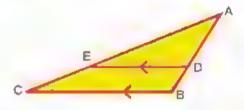


In the figure opposite :

ABC is an obtuse-angled triangle at B

DE // BC

Prove that: AE > AD

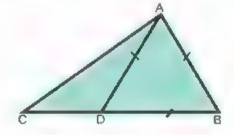


- 8 ABC is a thangle, CD bisects ∠C, CD ∩ AB = (D), prove that BC > BD
- ABC in which m (∠A) = $(5x + 2)^n$, m (∠B) = $(6x 10)^n$, m (∠C) = $(x + 20)^n$, order the lengths of the sides of the triangle ascendingly.

10 In the figure opposite :

ABC is a triangle, $D \in BC$, AB = AD = BD

Prove that : BC > AC



11 ABC is a right-angled triangle at B, D∈ AC, E∈ BC, where AD = BE

Prove that: m (∠CED) > m (∠CDE)

Lesson Four

Triangle inequality



Activity

By using your ruler and compass, try to draw the triangle ABC where :

$$\bigcirc$$
 AB=4cm, BC=5cm, AC=6cm

$$2AB = 6 cm$$
, $BC = 3 cm$, $AC = 2 cm$

$$3AB=9$$
 cm, $BC=4$ cm, $AC=3$ cm

$$\bigcirc$$
 AB = 8 cm, BC = 3 cm, AC = 5 cm

In which of the previous cases were you able to draw the triangle? What do you conclude?

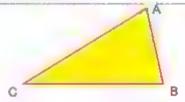


For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

i.e.: In any triangle ABC:

$$BC + CA > AB$$

AB +AC >BC



for example: the numbers 5, 3 and 9 are not valid to be the lengths of a triangle because the sum of the smallest two numbers = 3 + 5 = 8, 8 < 9. Therefore, the inequality of the triangle is not verified.



Examples

ABC is a triangle, If AB = 10 cm.

BC = 8.5 cm

Find the interval which the length of side AC belongs to.

You will learn how To define the triangle

 To define the triangle inequality.

key term/

to inequality.

tnangle inequality



Solution

triangle inequality

From
$$(1)$$
, (2) $18.5 > AC > 1.5$



Find the interval which the third side belongs to in each of the follwing triangles. If the lengths of the other two sides were as follows:

6 cm, 9 cm B 5 cm, 12 cm C 7 cm, 15 cm D 2.9 cm, 3.2 cm

Exercise (4-4)

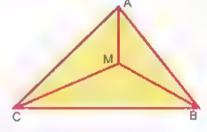
- If the lengths of two sides in an isosceles triangle were 5cm, 12cm, Find the length of the third side (State the reason)
- Which of the following groups are valid in drawing a triangle?

 - 🏇 5 cm, 7 cm, 8 cm 💮 4 cm, 9 cm, 3 cm

 - 10 cm, 6 cm, 4 cm 15 cm, 17 cm, 30 cm.
- Prove that the length of any side in a triangle is less than half of the perimeter of the same triangle.
- In the figure opposite:

ABC is a triangle in which M is a point inside it. Prove that:

MA + MB + MC > $\frac{1}{2}$ perimeter of the triangle ABC



prove that: the sum of the lengths of the two diagonals in a convex quadrilateral is less than its preimeter.

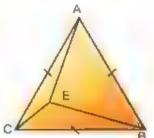


General Exercises

In the figure opposite: ABC is an equilateral thangle where E is a point inside it

m (ECB) > m (EBC).

First: prove that : m ($\stackrel{\frown}{ABE}$) > m ($\stackrel{\frown}{ACE}$). Second: m (A) > m ($\stackrel{\frown}{ABE}$) > m ($\stackrel{\frown}{ACE}$).

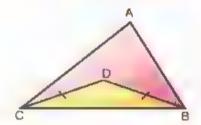


In the figure oppsoite :

D B = D C .

m (ABC) > m (ACB)

Prove that: m (ABD) > m (ACD)

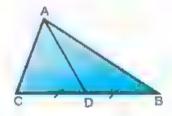


- A B C is a triangle in which AB = 6cm, A C ≠ 7cm, B C = 8cm.

 Order the measures of its angles ascendingly.
- un the figure opposite :

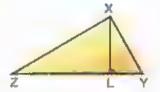
AB > AC , DB = DC

Prove that: m (BAD) < m (CAD).



In the figure opposite :

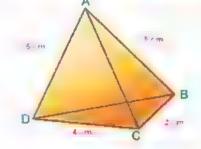
 $\frac{XZ>XY}{XL} \perp \frac{A}{ZY}$ prove that m(LXZ)>m(LXY)



6 In the figure opposite:

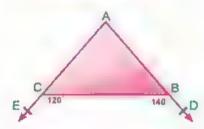
ABCD is a quadrilateral in which

Prove that: m (ABC) > m (ABC)



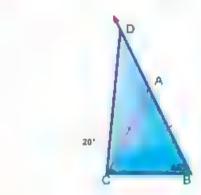
In the figure opposite :

Prove that: CB>AB



In the figure opposite:

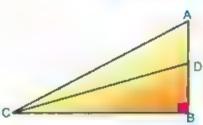
Prove that: AB > AD



In the figure opposite:

$$m(\hat{B}) = 90^{\circ}$$

Prove that: AC > DC

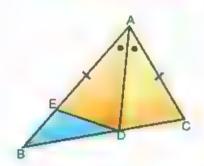


10 in the figure opposite:

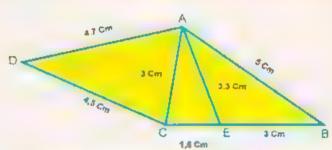
$$AC>DC, m(CAD)=m(BAD)$$



BD > DC .



Activity



- In the given figure complete using (< or >)
 - ♠ m (∠ D A C) m (∠A C D)
 - 6 m (ZA E C) m (ZE C A)
 - 🦫 m (∠A В Е) т (∠ Е А В)
 - ♠ m (∠C D A) m (∠D A C)
 - m (∠A E B) m (∠ E A C)
- In the traingle ABC, AB = 6cm, BC = 9cm

then AC ∈] ,

3 In the thangle ABC · m (\angle A) = (9x)°, m (\angle B) = (6x - 17)° m (\angle C) = (7 x - 1)°

Order the lengths of the sides of the thangle ascendingly.

(0)

Unit test



1 Complete the fllowing to make each statement true:

- A in a triangle, the smallest angle in measure is opposite to
- B In \triangle ABC: if m (\angle A) = 70°, m (\angle B) = 30°, then the longest side in length is ...
- C If the lengths of two sides in an isosceles triangle were : 3 cm, 7cm then , the length of the third side = cm
- D \triangle ABC in which: m (\angle A) = 100°, then the greatest side length is
- E Δ ABC in which AB = 3cm, BC = 5cm, then AC ∈] [
- F The longest side length in the right-angled triangle is

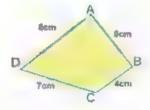
In the figure opposite :

ABCD is a quadrilateral in which

$$AB = 6cm$$
, $BC = 4cm$,

Prove that:

m (∠ BCD) > m (∠ BAD)

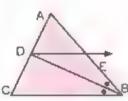


In the figure opposite:

A B C is a triangle, BD bisects \angle B, BD n AC = {D},

DE // CB and intersects AB at E

Prove that: AB>AD



🤞 In the figure opposite :

△ ABC in which AB > AC, D ∈ AB , E ∈ AC BF bisects ∠ DBC , CF bisects ∠ BCE

BF n CF = (F)

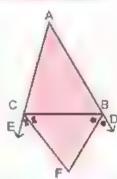
Prove that:



 $m (\angle FBC) > m (\angle BCF)$



CF > BF



Model (1)

First question : complete the following :

- (2) If the lowest boundary of a set is 10 and the upper boundary is X and its centre is 15, then X =
- $(3)]-2+2] (]\{-2+0\} =$
- (4) A cube whose volume is 8 cm³, then the sum of the lengths of all its edges equals cm.
- (5) The multiplicative inverse of the number $\sqrt{3} + \sqrt{2}$ is (in the simplest form).

Second question: Choose the currect answer from those given.

(1) If the radius of a sphere is 6 cm, then its volume is

a. 6 π cm³

b. 36 π cm³ c. 72 π cm³

d. 288 π cm³

(2) If the point (a,1) satisfies the relation x+y=5, then a=

(3)
$$\left(2\sqrt[3]{2}\right)^3 =$$

c. 16

d. 40

(4) The median of the values 34, 23, 25, 40, 22, 4 is

a. 22

b. 23

c. 24

d. 25

(5) If the anthmetic mean of the values $27.8 \cdot 16.24.6 \cdot k$ is 14 then k =

b. 6

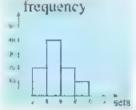
(6) In the opposite figure the value of the mode =

a. 4

h. 5

c.6

d.40



question (3)

- (a) Find the value of $\sqrt{18} + \sqrt[3]{54} 3\sqrt{2} \frac{1}{2}\sqrt[3]{16}$
- (b) If $\chi = \frac{3}{\sqrt{3} \sqrt{3}}$, $y = \sqrt{5} \sqrt{2}$, prove that: χ , y are two conjugate

question (4)

- (a) The area of a square is 1089 cm². Find the length of its diagonal.
- (b) Find the S.S of the inequality $\frac{3 \times 1}{6} < \times 1 < \frac{\times 4}{2}$ in \mathbb{R} , then represent it on the number line.



- (a) The radius length of the base of a right circular cylinder sits base radius length is 4
 - $4\sqrt{2}$ cm and its height is 9 cm find its volume in terms of π and if its volume equals the volume of a sphere find the radius length of the sphere.
- (b) Find the arithmetic mean of the following frequency distribution.

The set	5 –	15 –	25 –	35 -	45 –	total
Frequence	7	10	12	13	- 8	50

Model (2)

Ovestion one: Complete the following:

(1) The additive inverse of the number $-\sqrt{3}$ $-\sqrt{5}$ is

 $(2)(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2})=$

- (3) The conjugate of the number 2√5 -3√2 is . . .
 (4) If the volume a sphere is 9/2 π cm, then its diameter length is . . .
- $(5)\{3.4\}-\{3.5\}=$

Question (2): Choose the correct answer from those given:

(1) If the volume of a cube is 27 cm³ then the area of one of its faces is

a. 3 cm²

- b. 9 cm2
- c. 36 cm²
- a 54 cm2
- (2) If the mode of the set of values $4 \cdot 11 \cdot 8 \cdot 2 \times 11 \cdot 8 \cdot 2 \times 11 \cdot 10 = 10$

- a. 2 b. 4
- c. 6
- (3) If the arithmetic mean of the set of values 18, 23, 29, 2k-1, k is 18 then k =

a. I

- d. 90
- (4) If the lowest limit of a set is 4 and the upper limit is 8 then its centre is .

a. 2

- b. 4
- c. 6

- d. 8
- (5) A right circular cylinder, the radius of its base is r, its height equals the length of the dimeter, then its volume= ...

a. 2003

- d. 213
- (6) The sloution set in R of the equation $X(X^2-1) = 0$ is ...

a.{o}

- b.{1}
- c.{-1}
- d.{ 0,-1,1 }



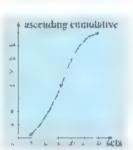
- (a) Reduce to the simplest form. $\sqrt{\frac{1}{5}} + \sqrt{\frac{5}{5}} + \sqrt{\frac{5}{5}} + \sqrt{\frac{5}{5}}$
- (b) Prove that $\sqrt[3]{128} + \sqrt[3]{16} 2\sqrt[3]{54} = 0$

question (4)

- (a) Find the S S of the inequality $-2 < 3 \times + 7 \le 10$ in $\mathbb{R}_{>}$ then represent the interval of solution on the number line.
- (b) If $\chi = \sqrt{2} + \sqrt{3}$ find the value of $\chi^4 \sim 2 \chi^2 + 1$

question (5)

(a) The opposite figure represents the ascending cumulative curve of the marks 32 students in one test complete the median mark =



(b) Find the arithmetic mean of the following frequency distribution

The set	5	15 -	25 -	35	45	total
Frequence	4	5	6	3	2	20

Model Test (3)

"Merge students"

Questions (1) Complete each of the following:

- 3) The mode for the number $\mathbf{3}$, $\mathbf{5}$, $\mathbf{3}$, $\mathbf{4}$, $\mathbf{3}$ is
- 4) The median of the values 2 , 3 , 5 , 7 , 9 is ..
- 5) The solution set of the equation $x^2 + 9 = 0$ in R is

Questions (2) Choose correct answer.

- 1) The arithmetic mean for the values 9, 6, 5, 14, 1 is
 - (a) 7
- (b) 3
- (c) 5
- (d) 9
- - (a) $\sqrt{3}$
- (b) 1
- (c)√2
- (d) 2 √3
- 3) The additive inverse of the number $\sqrt{5}$ is
 - (a)√5
- (b) 5
- (c) √2
- (d) -5

- 4) [3,5]-{3,5} =
 - (a) [3,5] (b) [3,5]
- (c) ø
- (d)] 3, 5]
- 5) A cube of volume 64 cm3, then the edge length is cm.
 - (a)4
- (b) 8
- (c) 16
- (d) 64

Questions (3) Match from the column (A) to the suitable one from the column (B):

(A)	(B)
1) The solution set of the equation $x^2 - 25 = 0$ R is	[0,2]
2) [-3,2] \cap [0,2] =	7
If the order of the median is the fourth, than the number of the values is	{5 , -5}
4) √3 Isan . number	3
5) The solution set of the inequality 3 ≤ x ≤ 7 on the number line is	Irrational

Questions (4) Put (1) for the correct statement, (x) for the incorrect one:

- 1) The arithmetic mean for the a set of values = the sum of these values + its number.
- 2) If $x = \sqrt{13} \sqrt{7}$, $y = \sqrt{13} + \sqrt{7}$, then x, y are two conjugate numbers.
- 3) The irrational number $\sqrt{7}$ lies between 2 and 3
- 4) $\sqrt{75} 2\sqrt{27} = 7\sqrt{3}$
- 5) The simplest from of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$

Questions (5) First: Complete:

Second : Complete

The following table to obtain the arithmetic mean of the following frequency distribution

Sets	5-	15-	25-	35-	45-	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set x	Frequency (F)	FxX
5 –	10	7	10 x 7 = 70
15 -	20	10	20 x 10 =
25 –	^	100 101 401 010 407 74	x 12 =
35 –	**************************************	*** ** ** *** *** *** *** ***	x 13 =
45 -	4 8 8 7 8 8 7 4 8 7 4 8 7 8 8 8 8 8 8 8 8 8 8	190414111111111111111111111111111111111	x8=
The sum		50	

The arithmetic mean = $\frac{\Sigma(F \times X)}{\Sigma(F)}$ = ______

Model tests on Geometry 2 nd preparatory

Model (1)

Complete the following :

The longest side in the right angled triangle is

If the lengths of two sides in a triangle are 2 cm, and 7 cm, then∠the length of the third side <∠

If the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length then

if the measure of an angle in the isosceles triangle equals 60° then the triangle is

Choose the correct answer from those given :

In the opposite figure:

 \triangle ABC is equilateral, then m (\angle ACD) =

- (a) 45°
- (b) 60°
- (c) 120°
- (d) 135°



In \triangle ABC which is right angled at B , if AC = 20 cm, then the length of the median of the triangle drawn from B equals

- (a) 10 cm.
- (b) 8 cm.
- (c) 6 cm.
- (d) 5 cm.

XYZ is a triangle in which m ($\angle Z$) = 70° and m ($\angle Y$) = 60° then YZXY

- Cars
- (h)
- (0) -
- (d) twice

The lengths which can be lengths of sides of a triangle are

- (a) (0 + 3 + 5)
- (b) (3 , 3 , 5)
- (e) (3 + 3 + 6)
- (d) (3 · 3 · 7)

The triangle in which the measure of two angles of it are 42° and 69° is

(a) an isoscles triangle.

(b) an equilateral triangle.

(c) a scalene triangle.

(d) a right angled triangle.

In the opposite figure, $m(\angle A) = 2m(\angle C)$, $AC = \dots cm$.

- (a) 3
- (b) 6
- (c) 9
- (d) 12



/a/ If \triangle ABC in which AB > AC then m (\angle C) m (\angle B)

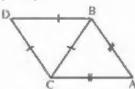


[b] In the opposite figure:

 $m (\angle A) = 50^{\circ} AB = AC$

and A DBC is an triangle equilateral

Find m (∠ ABD)

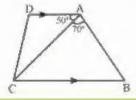


[c] In the opposite figure:

AD // BC + m (∠ BAC) = 70°

and m (\angle DAC) = 50°

Prove that : BC > AC



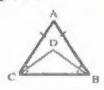


- [a] Prove that the base angles of the isosceles triangle are congruent.
 - (b) In the opposite figure:

AB = AC , BD bisects & B and CD

bisects & C

Prove that: A DBC is isosceles



[a] In the opposite figure:

Arrange the angles of A ABC descendingly

due to their measures



[b] In the opposite figure:

AB > BC , XY // BC

Prove that :

AX > XY





- Choose the correct answer from those given :
 - 1. The triangle which has three axes of symmetry is

(a) scalene

(b) isosceles

(c) right angled

(d) equilateral.

2. The sum of lengths of two sides in a triangle is the length of the third side.

(a) greater than

(b) smaller than

(c) equals to

(d) twice

If the lengths of two sides in an isosceles triangle are 8 cm, and 4 cm, then the length of the third side is

(a) 4

(c) 3

(d) 12

(b) AC

(c) AB

(d) its median

5. △ XYZ is an isosceles triangle in which m (∠ X) = 100° then m (∠ Y) =

(a) 100

(b) 80

(c) 60

6. In the opposite figure x+y =

(a) 100

(b) 140

(c) 180

(d) 280

2 Camplete the following:

- 1. If the measure of an angle in a right angled triangle is 45° then the triangle is
- 2. The length of any side in a trianglethe sum of lengths of the (wo other sides.
- 3. If AB = XY then AB =
- In △ ABC of m (∠ A) = 30° and m (∠ B) = 90° then BC =AC
- 5. The axis of symmetry of a line segment is the straight line which at its midpoint.
- [a] In \triangle ABC in which AB = 7 cm. > BC = 5 cm. and AC = 6 cm. Arrange its angles ascendingly due to their measure.

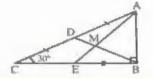
[b] In the opposite figure:

ABC is right angled at B .

 $m (\angle C) = 30^{\circ} \cdot D$ is the midpoint of AC

 \bullet E is the midpoint of BC \bullet AC = 9 cm.

Find the length of each of BD , BM and AB



[a] In the opposite figure :

$$m (\angle ABC) = m (\angle BDE) = 90^{\circ}$$

$$m (\angle E) = 30^{\circ}$$

D is the midpoint of AC

Prove that : AC = BE

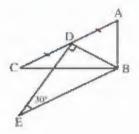
[b] In the opposite figure:

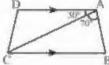
$$\overline{AD} // \overline{BC} \cdot m (\angle BAC) = 70^{\circ}$$

• m (∠ DAC) = 30°

Prove that:

AC > BC



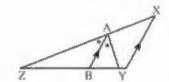


[b] In the opposite figure:

AB // XY and AB bisects & YAZ

Prove that :

XZ > YZ



Model Test

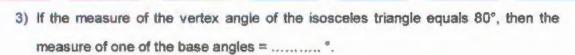
" Merge students "

Questions (1) Complete:

- In the right-angled triangle, the length of the median drawn from of the right angle equals
- 3) The base angles of the isosceles triangle are
- 4) In ∆ ABC, m (∠B) = 70°, m (∠C) = 50°, then AC AB
- 5) The median of an isosceles triangle from the vertex angle

Questions (2): Choose the correct answer:

- - (a) 30°
- (b) 60°
- (c) 70°
- (d) 90°
- - (a) $\frac{1}{2}$
- (b) $\frac{1}{3}$
- (c) $\frac{1}{4}$
- (d) 2



- (a) 60
- (b) 40
- (c) 30
- (d) 50

- (a) 1
- (b) 2
- (c) 3
- (d) 0

5cm

5)
$$\triangle$$
 ABC, m (\angle A) = 50°, m (\angle B) = 60°, then the longest side is

- (a) AB
- (b) BC
- (c) AC

Questions (3): In the opposite figure:

ABC is right-angled triangle at B,

m (∠C) = 30°, AB = 5 cm.

Find the length of AC

$$AB = \frac{1}{2} \times \dots$$

Questions (4):

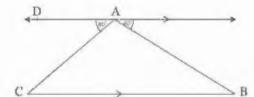
A) \triangle ABC, m (\angle A) = 40°, m (\angle B)= 75°, m (\angle C) = 65°, arrange the lengths of the sides of this triangle descendingly the order is:

C 30"

B) In the opposite figure:

Complete:

2- The longest side of A ABC is



Questions (5): In the opposite figure Put () infront of the correct answer. (X) for the incorrect answer:

AB = AC = CD = AD = 10 cm , m (BAC) = 70°

- 1) m (\angle B) = 55°
- 2) m ($\angle D$) = 70°
- 3) m (\angle DCB) = 120°
- ()
- 4) AB + AD = 20 cm
- ()
- 5) AB + BC = BC + CD
- (
- 7